UMTYMP Geometry Day 2

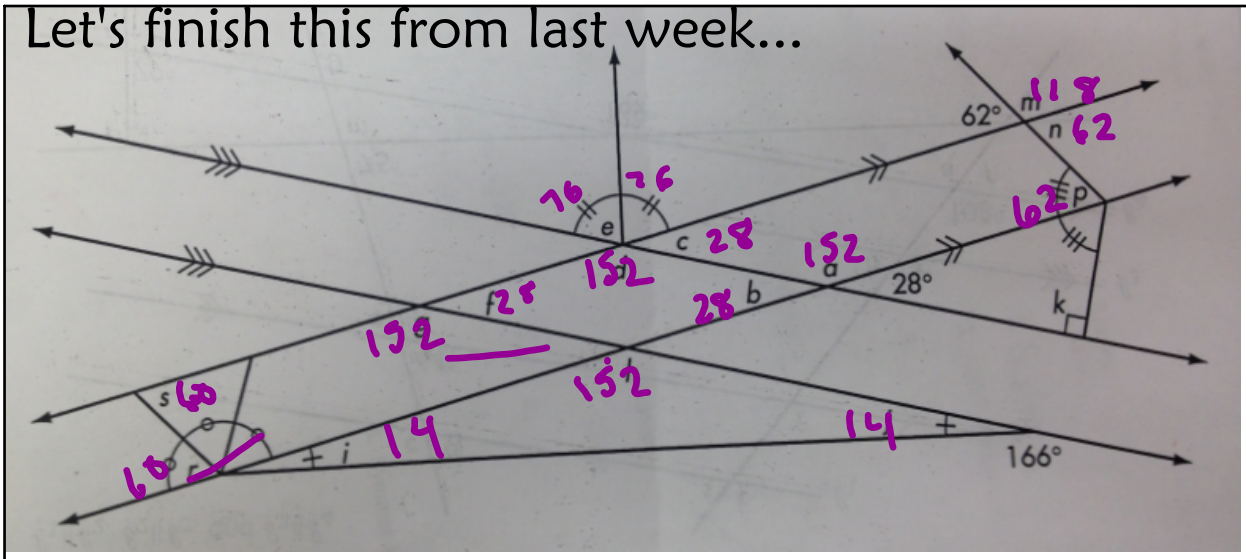
Congruent Triangles

Vincent Hall Room 6

Instructor: Andrea Butler

https://www.mathopenref.com/congruentssa.html

Let's finish this from last week...



a= b= c= d= e= f= g= h=
i= j= k= m= n= p= r= s=

Reminders about Constructions:

Sketch:

Freehand

Draw:

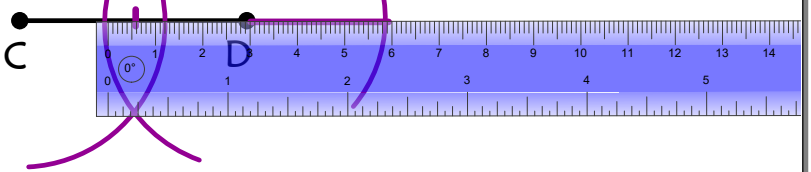
Ruler and Protractor

Construct:

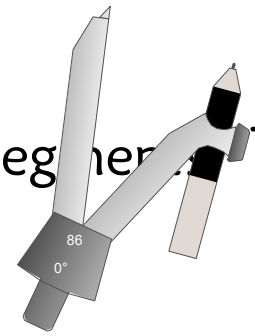
Compass and Straightedge

Duplicate a segment:

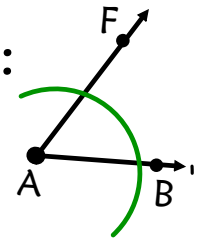
Find a midpoint:

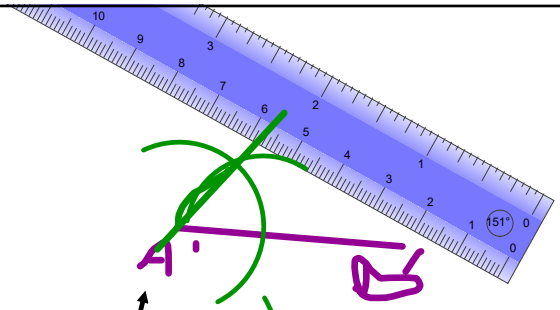


Add segments: $\overline{CD} + \overline{AB}$

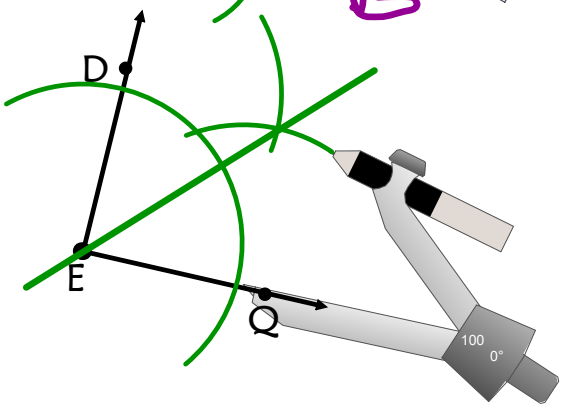


Duplicate angles:

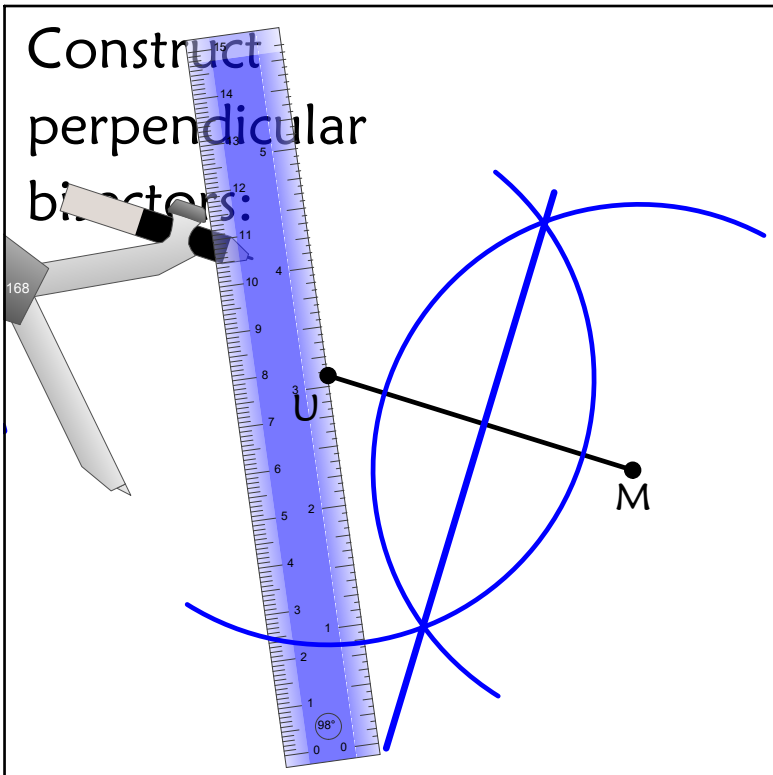




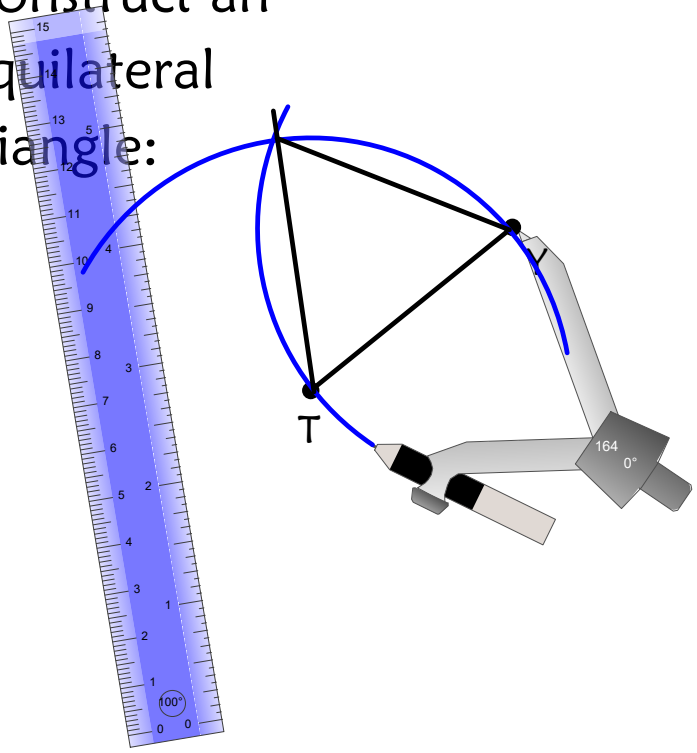
Construct angle bisectors:



Construct perpendicular bisectors:

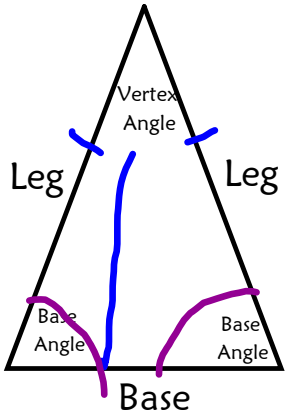


Construct an equilateral triangle:

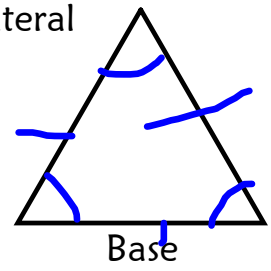


Triangles:

Isosceles



Equilateral



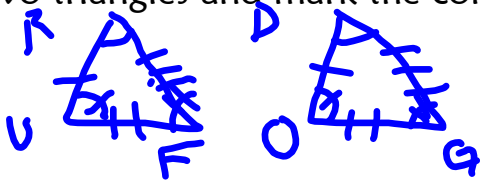
If $\Delta RUF \cong \Delta DOG$, which side is congruent to DG?

If $\Delta RUF \cong \Delta DOG$, which side is congruent to UF?

If $\Delta RUF \cong \Delta DOG$, which angle is congruent to angle O?

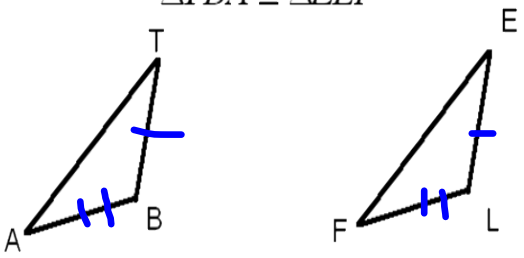
If $\Delta RUF \cong \Delta DOG$, which angle is congruent to angle GDO?

Sketch an example of these two triangles and mark the congruencies.

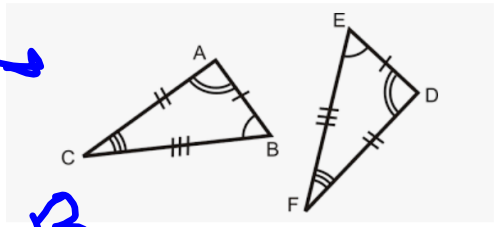


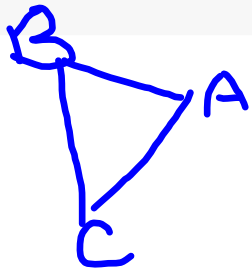
State every congruence and label them on the figures.

$\Delta TBA \cong \Delta ELF$

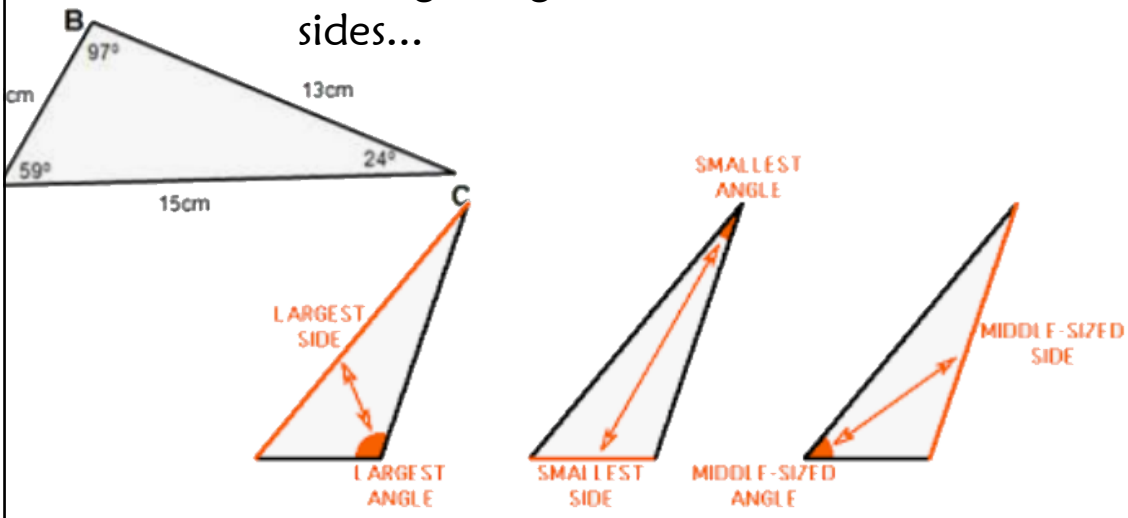


State the parts that we know to be congruent.





One more important relationship to know about triangle angles and sides...

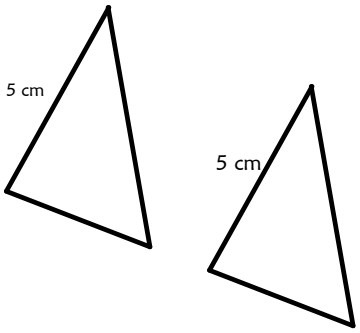


https://www.mathopenref.com/trianglesideangle.html

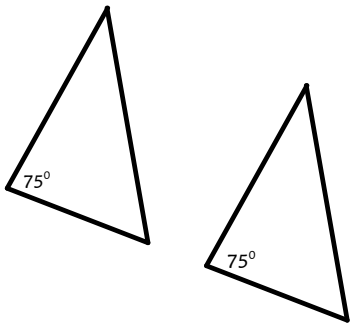
How do we know when two triangles are congruent?

Can we prove triangle congruence with just one piece of information?

One Side Congruent



One Angle Congruent

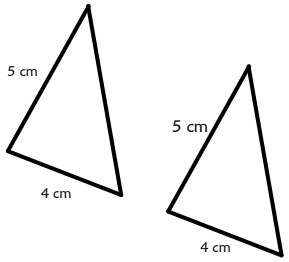


This never proves congruency.

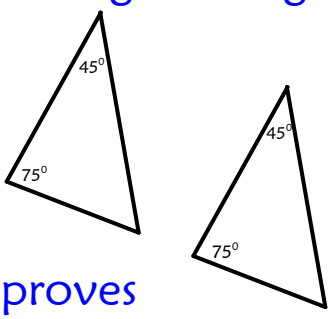
How do we know when two triangles are congruent?

Can we prove triangle congruence with two pieces of information?

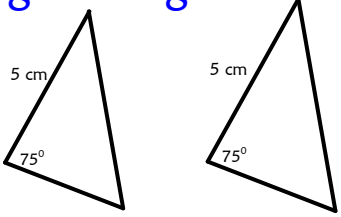
Two Sides Congruent



Two Angles Congruent



One Side and One Angle Congruent

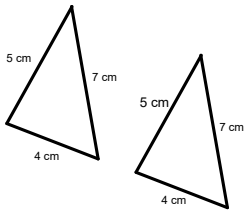


This never proves congruency.

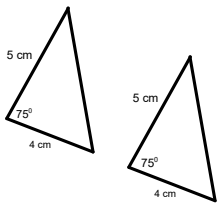
How do we know when two triangles are congruent?

Can we prove triangle congruence with three pieces of information?

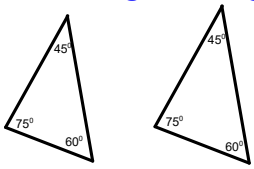
Three Sides Congruent



Two Sides and an Angle Congruent



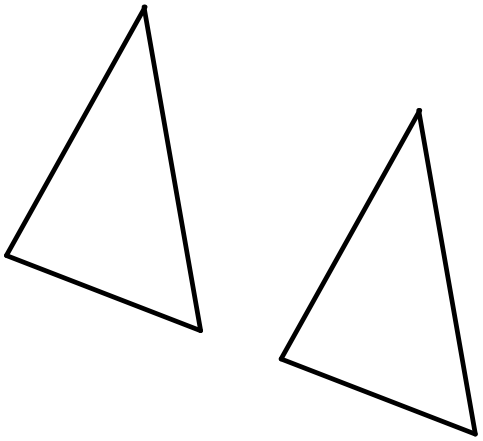
Three Angles Congruent



This sometimes proves congruency. We will go over which ways work and which don't today.

How do we know when two triangles are congruent?

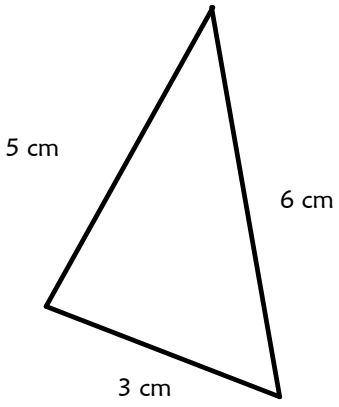
Can we prove triangle congruence with four pieces of information?

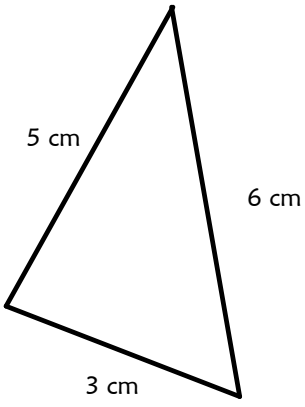


This always proves congruency.

How do we know when two triangles are congruent?

Does 3 sides prove triangle congruence?





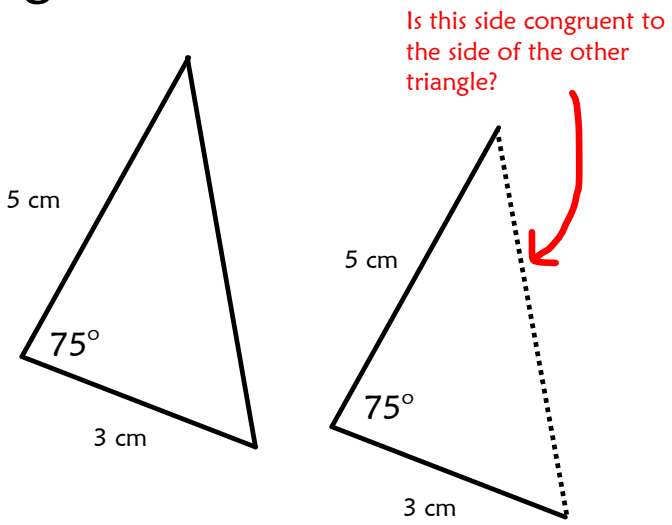
SSS - (side-side-side)

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

Draw a picture to show this.

How do we know when two triangles are congruent?

Does 2 sides and an angle prove congruence?



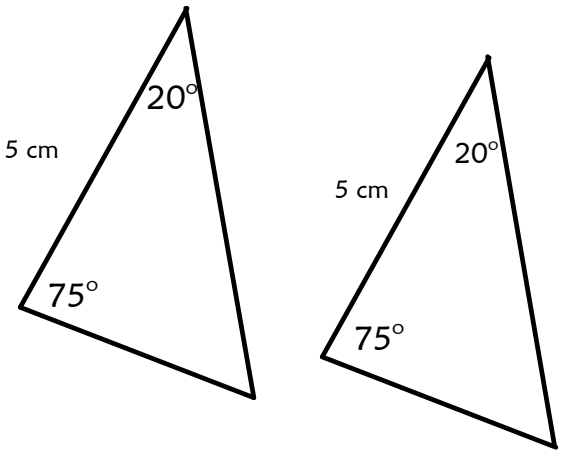
SAS - (side-angle-side)

If two sides and the angle between them in one triangle are congruent to two sides and the angle in between them in another triangle, then the triangles are congruent

Draw a picture to show this.

How do we know when two triangles are congruent?

Does 2 angles and a side prove congruence?



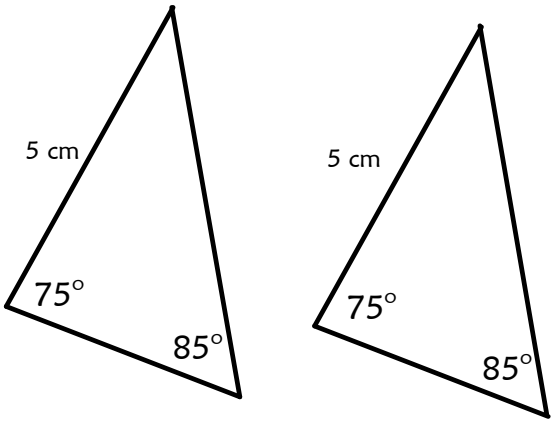
ASA - (angle-side-angle)

If two angles and the side between them in one triangle are congruent to two angles and the side between them in another triangle, then the triangles are congruent.

Draw a picture to show this.

How do we know when two triangles are congruent?

Does 2 angles and a side prove congruence?

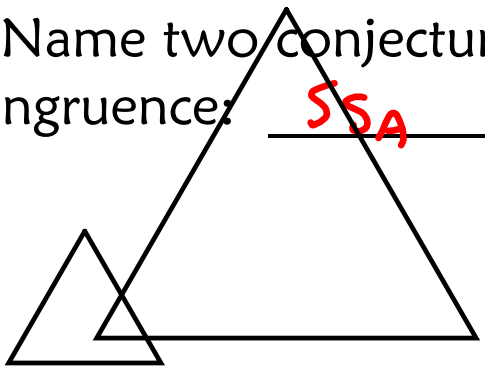


SAA - (side-angle-angle)

If two angles and a side that is not between them in one triangle are congruent to the corresponding two angles and side not between them in another triangle, then the triangles are congruent.

Draw a picture to show this.

- Name two conjectures that do not prove congruence: SSA and AAA.

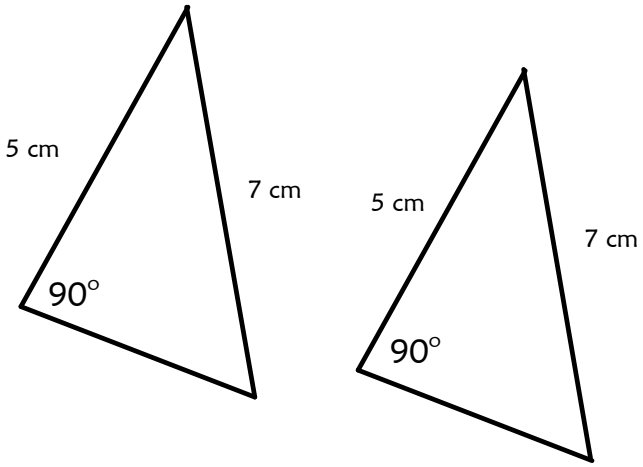


https://www.mathopenref.com/congruentssa.html

How do we know when two triangles are congruent?

There is one time where SSA always works...

But the angle needs to be 90 ^{0}.



HL - (Hypotenuse-Leg)

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

Draw a picture to show this.

How can we prove that the other parts of congruent triangles are congruent?

CPCTC-

Corresponding

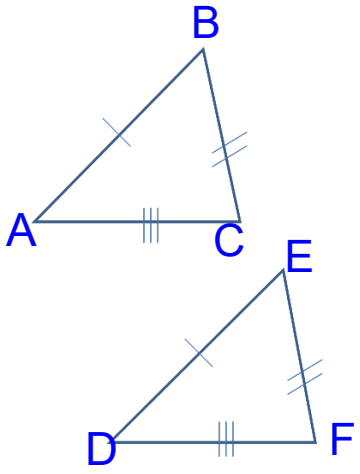
Parts of

Congruent

Triangles are

Congruent

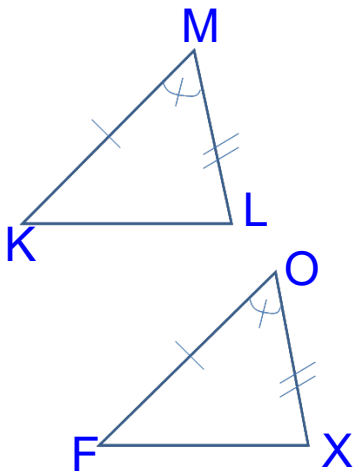
Name the Congruence Conjecture and state the congruent triangles



- A) SSS
- B) SAS
- C) SSA
- D) ASA
- E) SAA
- F) AAA

△ _____ ≅ △ _____

by _____

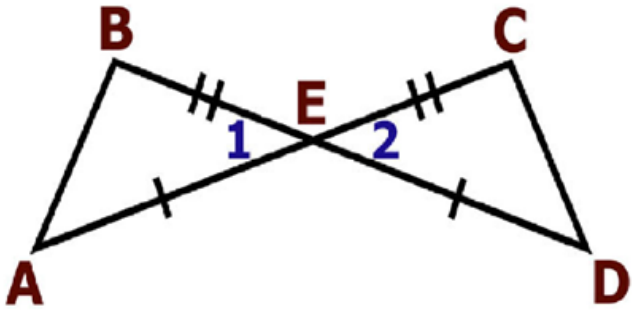


- A) SSS
- B) SAS
- C) SSA
- D) ASA
- E) SAA
- F) AAA

△ _____ ≅ △ _____

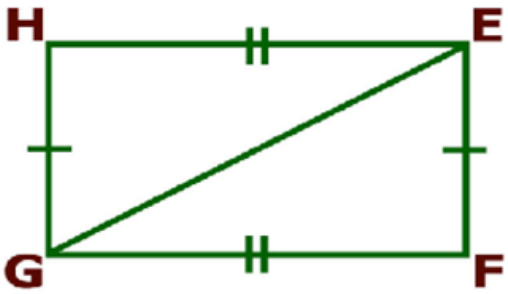
by _____

- A) SSS
- B) SAS
- C) SSA
- D) ASA
- E) SAA
- F) AAA



△ _____ ≅ △ _____

by _____



- A) SSS
- B) SAS
- C) SSA
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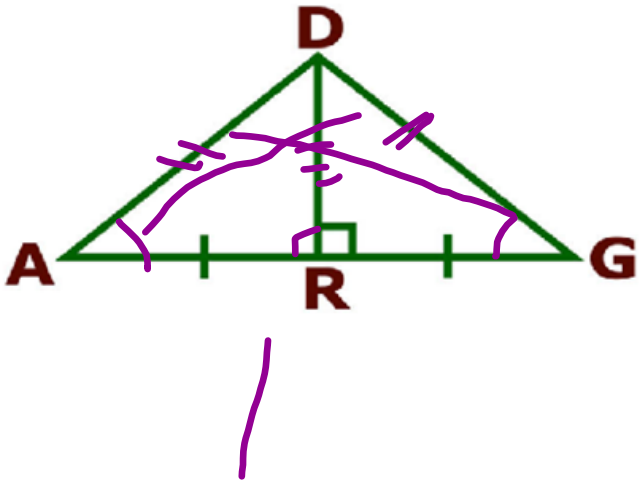
- F) AAA

△ _____ ≅ △ _____

by _____



- A) SSS
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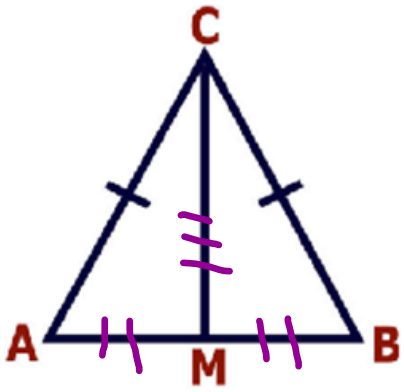
△ _____ ≅ △ _____

by _____



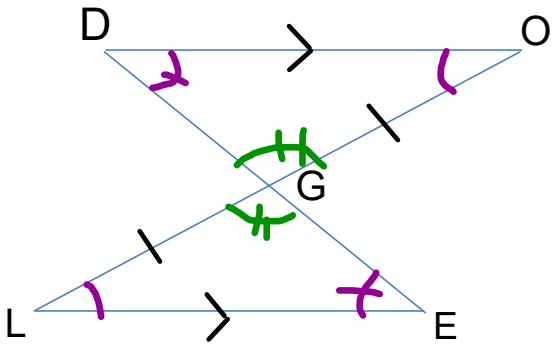
M is the midpoint of AB.

- A) SSS
- B) SAS
- C) SSA
- D) ASA
- E) SAA
- F) AAA

△ _____ ≅ △ _____

by _____

Which congruence conjecture fits the figures below?



- A) SSS
- B) SAS
- C) SSA
- D) ASA
- E) SAA
- F) AAA

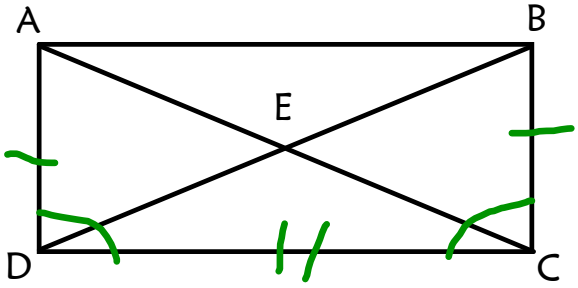
△ _____ ≅ △ _____

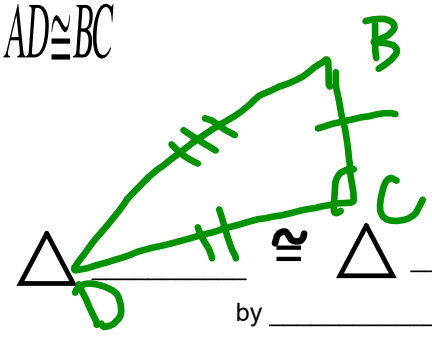
by _____

Given:

$\angle ADC \cong \angle BCD$

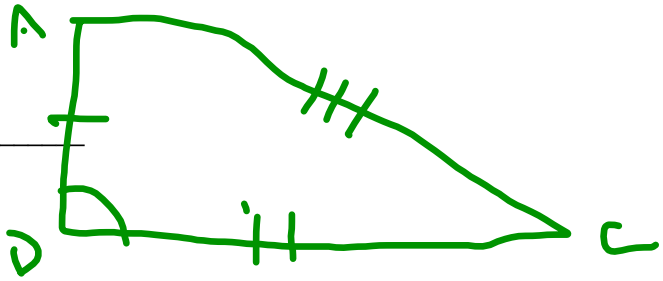
$\overline{AD} \cong \overline{BC}$

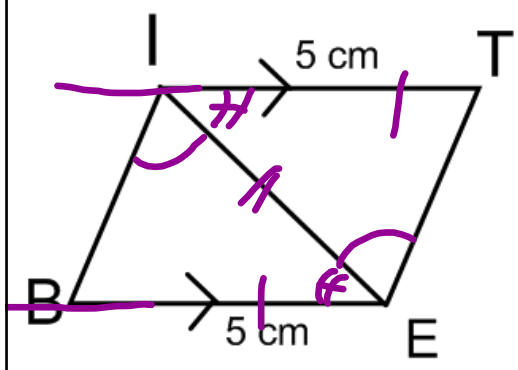




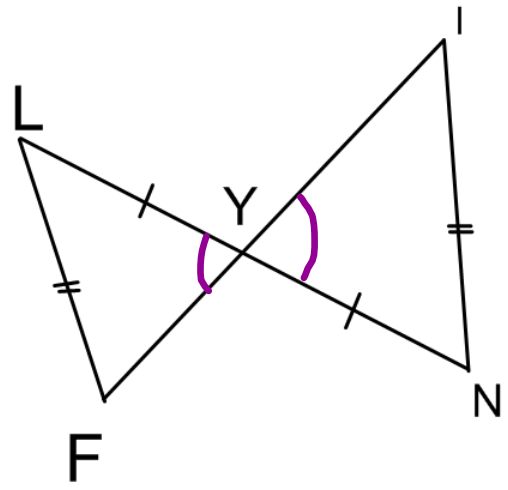
△ _____ ≅ △ _____

by _____





△ ___ ≅ △ ___ by ___



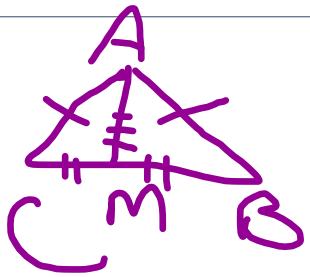
△ ___ ≇ △ ___ by ___

3.6.2:

Prove that if $AB = AC$ in $\triangle ABC$, then $\angle ABC = \angle ACB$. (Note: You cannot simply state that the triangle is isosceles, so the base angles are equal. You are asked here to prove this fact.)

Type your solution, notes and/or work here.

Submit



3.6.4:

Two angles of an equilateral triangle have measures $3x + 27°$ and $2y - 4°$. Find $x + y$.

Type your solution, notes and/or work here.

Submit

3.7.2:

Is it necessary when constructing the perpendicular bisector of $\overline{AB}$ to use circles with radius AB? Suppose we instead draw two intersecting circles with centers A and B and the same radius. Is the segment connecting the two points where these circles meet the perpendicular bisector of $\overline{AB}$?

Type your solution, notes and/or work here.

Submit

3.7.3:

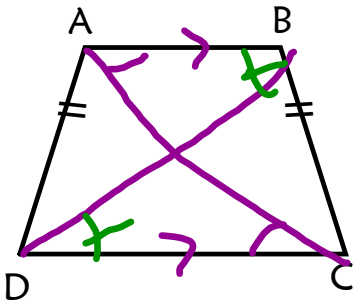
Construct a $90°$ angle.

Hint

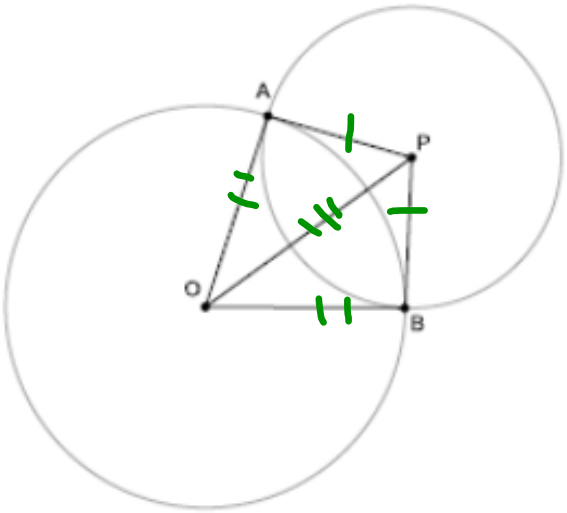
Type your solution, notes and/or work here.

Submit

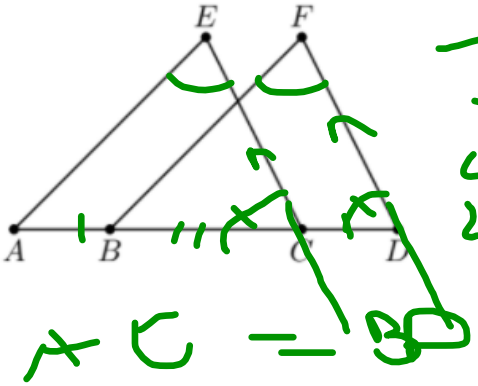
Given: Isosceles Trapezoid ABCD with bases AB and CD.
Prove: AC ≅ BD.



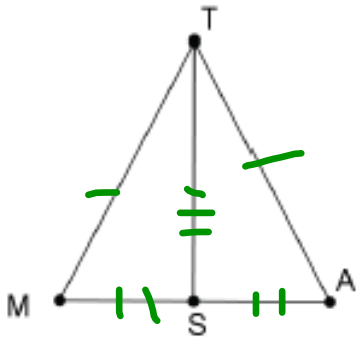
Given: Figure with two circles intersecting at A and B.
Prove: $\triangle OPA \cong \triangle OPB$



Given: Figure with $\angle AEC \cong \angle BFD$, $AB = CD$ and $\overline{EC} \parallel \overline{FD}$
Prove: $\triangle AEC \cong \triangle BFD$

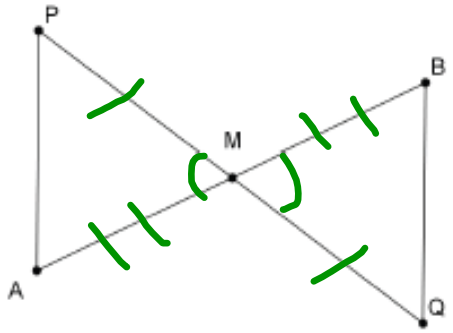


a. Given: $\overline{TS}$ bisects $\overline{MA}$
$\overline{MT} \cong \overline{AT}$



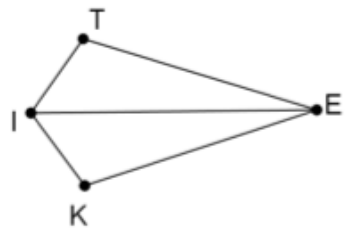
$\Delta MST \cong \Delta AST$ by _____ conjecture

a. Given: M is the midpoint of $\overline{AB}$ and $\overline{PQ}$



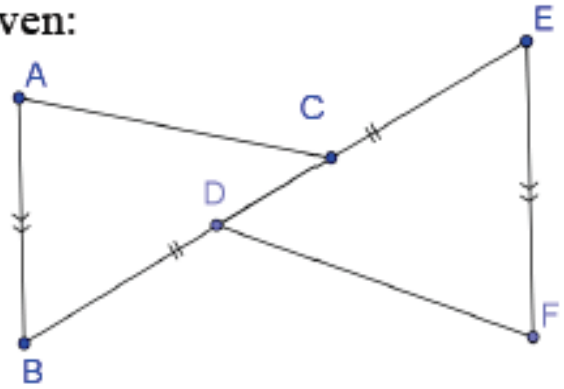
$\Delta APM \cong \Delta$_____ by _____ conjecture

b. Given:KITE is a kite with KI = TI and TE = EK



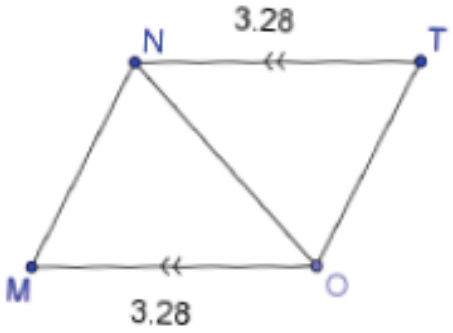
$\Delta ITE \cong \Delta$_____ by _____ conjecture

c. Given:



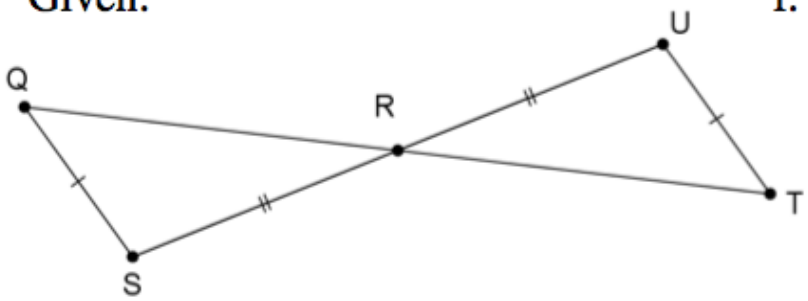
$\Delta ABC \cong \Delta$ _____ by _____ conjecture

d. Given:



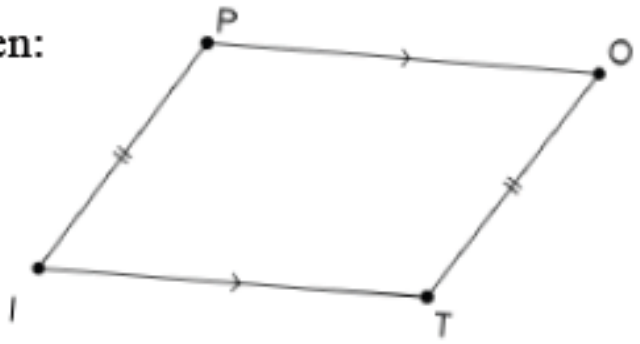
$\Delta MON \cong \Delta$ _____ by _____ conjecture

e. Given: f.



$\Delta SQR \cong \Delta$ _____ by _____ conjecture

f. Given:

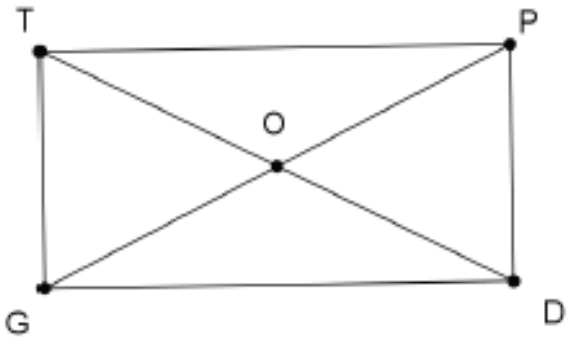


$\Delta PIT \cong \Delta$ _____ by _____ conjecture

g. Given: TPDG is a rectangle

$\overline{TO} \cong \overline{DO}$

$\overline{GO} \cong \overline{OP}$

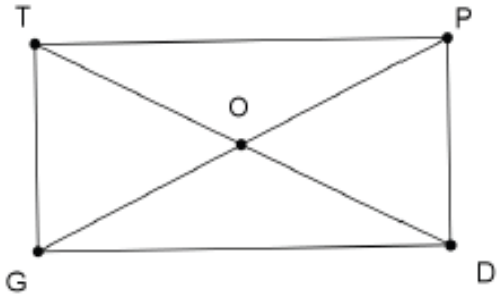


$\Delta TOP \cong \Delta$_____ by _____ conjecture

h. Given: TPDG is a rectangle

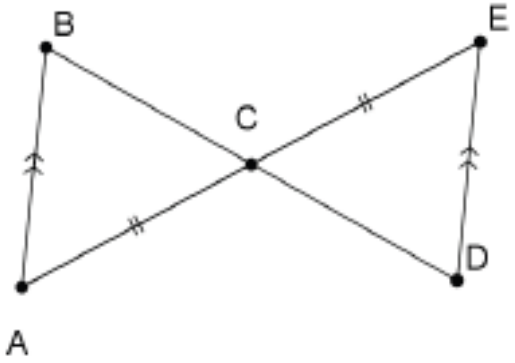
$\overline{TG} \cong \overline{PD}$

$\angle TGD \cong \angle PDG$



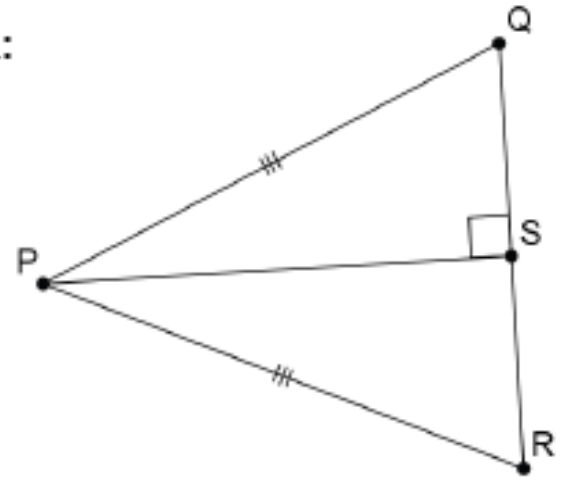
$\Delta TGD \cong \Delta$_____ by _____ conjecture

i. Given:



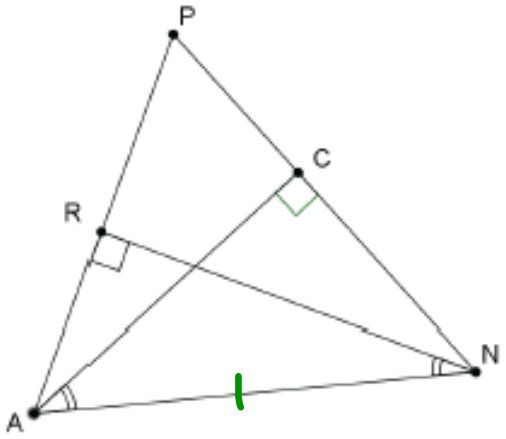
$\Delta ECD \cong \Delta$_____ by _____ conjecture

j. Given:



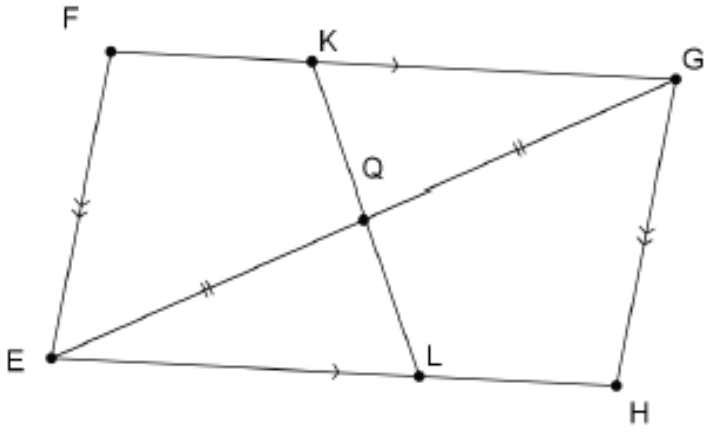
$\Delta PQS \cong \Delta$_____ by _____ conjecture

k. Given:



$\Delta ACN \cong \Delta$ ____ by _____ conjecture

l. Given: EFGH is a parallelogram



$\Delta EQL \cong \Delta$ ____ by _____ conjecture

Congruence Conjectures

First lets remind ourselves what congruent means. From your book:

Two figures are congruent if they are exactly the same — in other words, we can slide, spin, and/or flip one figure so that it is exactly on top of the other figure.

Triangle Congruence closing thoughts...

From your book:

- In more complicated geometry problems, mark side and angle equalities as you find them (particularly when you find non-obvious ones!)
- Dividing isosceles triangles in half by drawing a segment from the vertex between the equal sides to the midpoint of the base can be very effective.
- If you're stuck on an angle problem, assign one of the angle measures a variable and find other angles in terms of that variable. Hopefully, you'll eventually be able to build an equation you can use to solve for the variable.
- Mark the information you have in a problem on your diagram, particularly equal sides and equal angles. This will make congruent triangles particularly easy to find.
- Always be thinking about what you already know how to do when trying something new!

Triangle Congruence closing thoughts (con't.)

From your book:

- SSA (Side-Side-Angle) is not a valid congruence theorem. If two sides of one triangle are equal to two sides of another, and the two triangles have equal corresponding angles that are not the angles between the equal corresponding sides, then the two triangles are not necessarily congruent!