UMTYMP Geometry Day 3

Perimeter and Area

Vincent Hall Room 6

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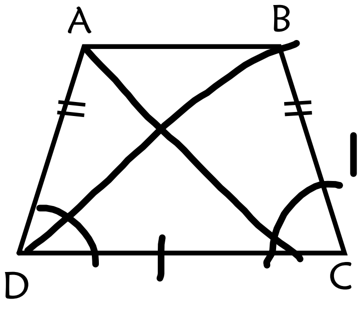
HW Collection
Quiz info??
Other concerns
Another shot at Proofs (15 min)
Lesson on Area and Perimeter

Let's finish this from last week...

Given: Isosceles Trapezoid ABCD with bases AB and CD.
Prove: AC ≅ BD.



Given Show

1. Given Isos Trap ABCD Bases AB | CD — Given

2. $\overline{AD} \cong \overline{BC}$ — 2. Given

3. Let $\overline{AC}$ & $\overline{BD}$ — 3.

4 DC ≅ DC — 4 Reflexive

5. ∠ADC ≅ ∠BCD — 5. Base ∠s isos trap ≅

CPCTC

Let's finish this from last week...

Two Column proofs

We'll do one together. Then you will

work on your own for a few minutes then check with a partner/group.

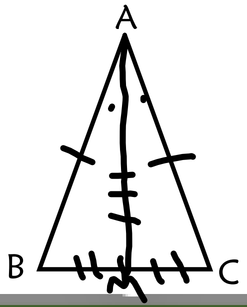
We do:

The isosceles triangle theorem states that if two sides of a triangle are congruent, then the angles opposite the sides are congruent.

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$

$m\angle A$



Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. Construct a bisector of $\angle A$	2. Every $\angle$ has 1 $\angle$ bisector
3. $\angle BAD \cong \angle CAD$	3. Definition of $\angle$ bisector
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\Delta ABD \cong \Delta ACD$	5. SAS Postulate
6. $\angle B \cong \angle C$	6. CPCTC

Congruence Conjectures

First lets remind ourselves what congruent means. From your book:

Two figures are congruent if they are exactly the same — in other words, we can slide, spin, and/or flip one figure so that it is exactly on top of the other figure.

Triangle Congruence closing thoughts...

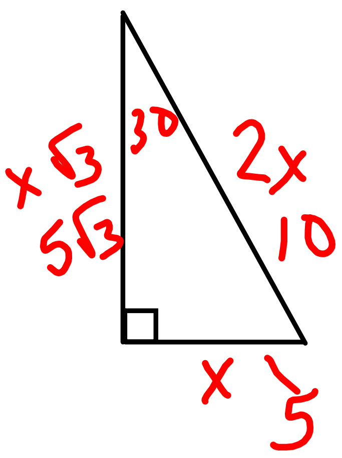
From your book:

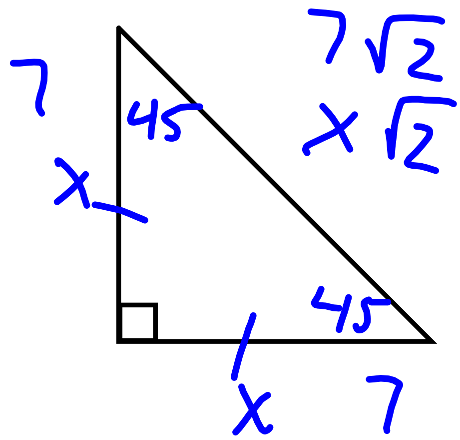
- In more complicated geometry problems, mark side and angle equalities as you find them (particularly when you find non-obvious ones!)
- Dividing isosceles triangles in half by drawing a segment from the vertex between the equal sides to the midpoint of the base can be very effective.
- If you're stuck on an angle problem, assign one of the angle measures a variable and find other angles in terms of that variable. Hopefully, you'll eventually be able to build an equation you can use to solve for the variable.
- Mark the information you have in a problem on your diagram, particularly equal sides and equal angles. This will make congruent triangles particularly easy to find.
- Always be thinking about what you already know how to do when trying something new!

New material this week:
Special Right Triangles

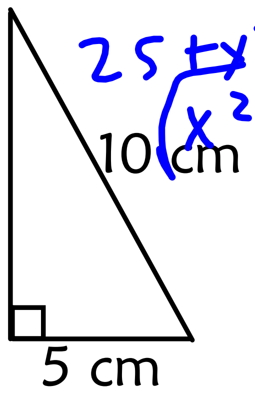
You need to KNOW they are 30-60-90 or 45-45-90 to use these rules.

30-60-90 45-45-90





Why do these work?



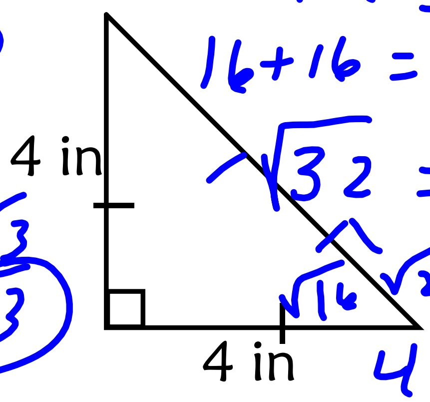
$5^2+x^2=10^2$

$25+x^2=100$

$\sqrt{x^2}=\sqrt{75}$

$\sqrt{25}\sqrt{3}$

$5\sqrt{3}$



$4^2+4^2=c^2$

$16+16=c^2$

$\sqrt{32}=\sqrt{c^2}$

$\sqrt{16}\sqrt{2}$

$4\sqrt{2}$

Perimeter-distance around
cm, ft, miles

Area-space inside 2 dimensional
cm^2, ft^2, square miles

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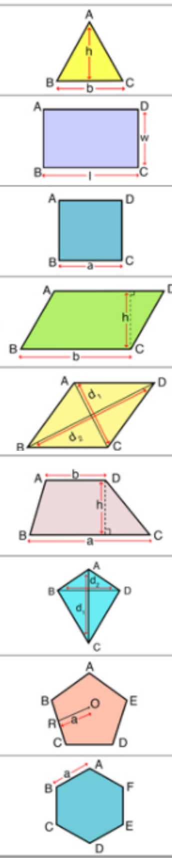
cm, ft, miles

Area-space inside 2 dimensional
cm^2, ft^2, square miles
[ABC] means the area of ΔABC

Common area formulas:
(you probably know most of these...)

Common area formulas



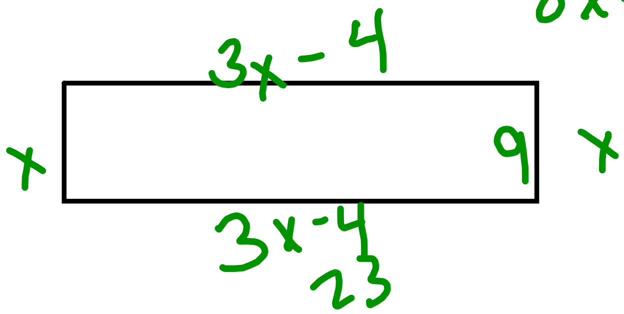
Polygons	Shapes	Formulas
Triangle		Area (A) = $\frac{1}{2}$(b × h) *here,* b = base, h = height
Rectangle		Area (A) = w × l *here,* w = width, l = length
Square		Area (A) = a^2 *here,* a = side
Parallelogram		Area (A) = b × h *here,* b = base, h = height
Rhombus		Area (A) = $\frac{d_1 \times d_2}{2}$ *here,* d_1 and d_2 are the diagonals
Trapezoid		Area (A) = $\frac{1}{2}$(a + b) x h *here,* a = long base b = short base h = height
Kite		Area (A) = $\frac{d_1 \times d_2}{2}$ *here,* d_1 and d_2 are the diagonals
Pentagon		Area (A) = $\frac{1}{2}$(p x a) *here,* p = perimeter a = apothem
Hexagon		Area (A) = $\frac{3\sqrt{3}}{2}$ x $(a)^2$ *here,* a = side

The ~~base~~ is also referred to as the altitude (next slide)

The length of one side of a rectangle is 4 less than 3 times and adjacent side. The perimeter of the rectangle is 64. Find the area.

$8x - 8 = 64$

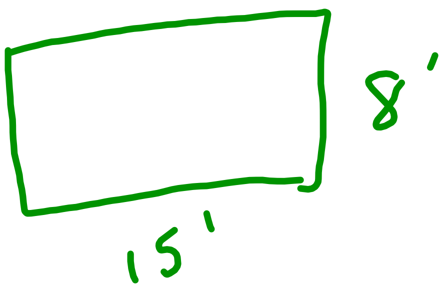
$x = 9$



$23 \cdot 9$

207 units^2

One gallon of paint covers 75 square feet of wall. How many gallons of paint do I have to buy if I paint two walls (no doors, windows, or closets) that are 15' long and 8' high?



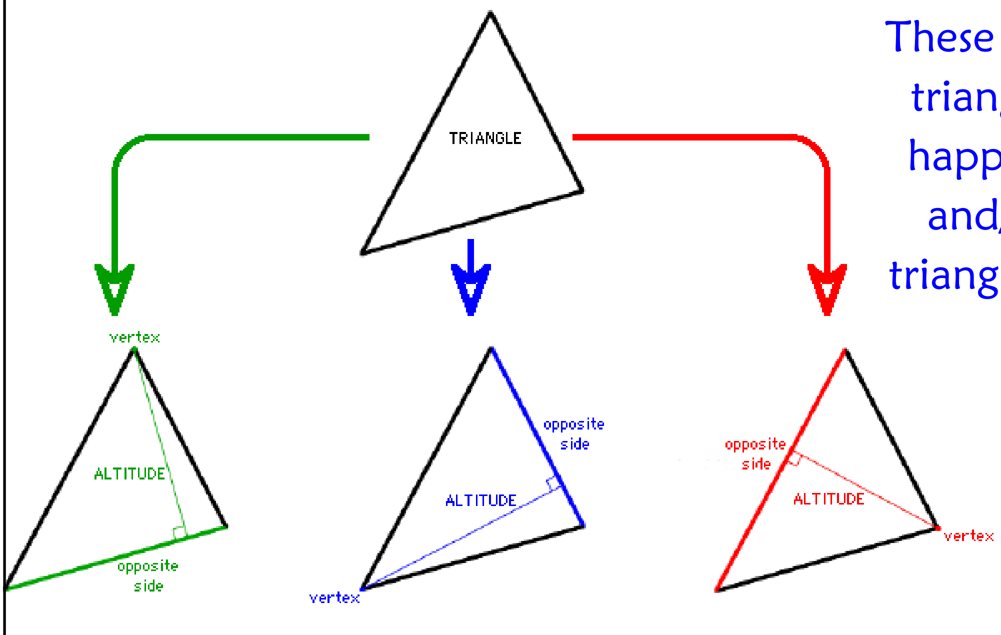
120 ft^2

$\times\ 2$

$\frac{240}{75} = 3.2$

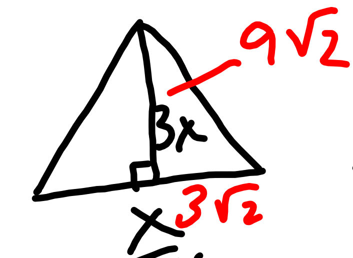
An altitude of a triangle is a segment from a vertex of the triangle, perpendicular to the side opposite (or a line containing the side opposite) that vertex of the triangle.

Since all triangles have three vertices and three opposite sides, all triangles have three altitudes.



These are all acute triangles. What happens in right and/or obtuse triangles? (click for link)

The area of a triangle is 27ft^2. If the height is 3 times the length of its base, find the height and base of the triangle.



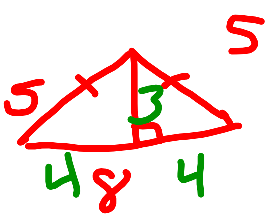
$$\frac{b\cdot h}{2} \qquad \frac{x\cdot 3x}{2} = 27$$

$$\frac{1}{2}bh$$

$$x\cdot 3x = 54$$
$$3x^2 = 54$$
$$x^2 = 18$$

$$x = \sqrt{18} = \sqrt{9}\,\sqrt{2} = 3\sqrt{2}$$

Find the [ABC] with side lengths of 5, 5, 8.



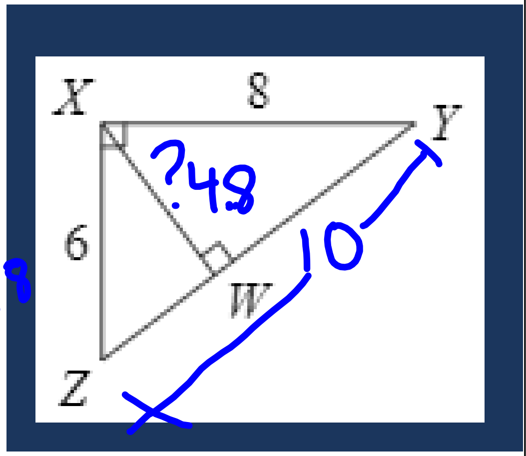
$$\frac{b\cdot h}{2}$$

$$\frac{8\cdot 3}{2} = 12$$

Given that $<ZXY = <XWY = 90^\circ$

and $XZ = 6, XY = 8, ZY = 10,$

Find XW



$\frac{1}{2}\cdot 6\cdot 8 = 24$

$\frac{XW\cdot 10}{2} = 24$

$10\cdot XW = 48$

$XW = 4.8$

Find the area of an equilateral triangle with a perimeter of 24 in.

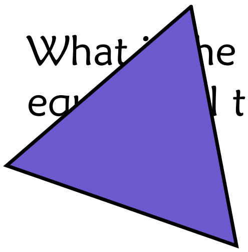
$P = 24$ in

8 8 $4\sqrt{3}$ 60 60 8 4

$\frac{8\cdot 4\sqrt{3}}{2}$

$\frac{32\sqrt{3}}{2}$

$16\sqrt{3}$ in^2



What is the simplified area formula for any equilateral triangle?

$\frac{1}{2}a\sqrt{3}$ a a a

$2x\sqrt{3}$

$\frac{x^2\sqrt{3}}{4}$

$A = \frac{\sqrt{3}}{4}a^2$ Where A = area and a = side length

$\frac{a^2\sqrt{3}}{4}$

$\frac{a\sqrt{3}}{2}\cdot\frac{a}{2}$

$\frac{8^2\sqrt{3}}{4} = 16\sqrt{3}$

2 2 $\sqrt{3}$ 2 1

$\frac{2\cdot\sqrt{3}}{2}$

Same base, same altitude...

(from pg. 89 in your book)

When you have an altitude in common...the ratio of areas of the triangles equals the ratio of the lengths of the sides to which this common altitude is drawn.

Same base, same altitude (con't)...

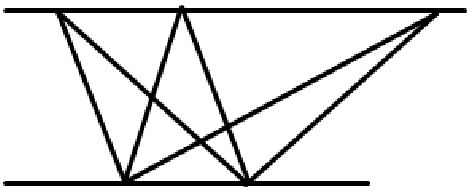
(from pg. 90 in your book)

> If two triangles share an altitude, then the ratio of their areas is the ratio of the bases to which that altitude is drawn. This is particularly useful in problems in which two triangles have bases along the same line.

> If two triangles share a base, then the ratio of their areas is the ratio of the altitudes to that base.

Same Base/Same Altitude

Prove that the diagram below has 3 triangles with same area (same base, altitude, // lines)



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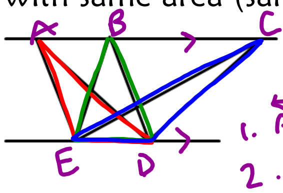

Statement	Reason
1. $\overleftrightarrow{AC} \parallel \overleftrightarrow{ED}$	1. Given
2. All △s have Base $\overline{ED}$	2. Given
3. lines are same distance apart	3. Defn //
4. Base on // line, altitude ⊥ to // Since lines same dist. apart, all altitudes are same	
5. $A_\triangle = \frac{b \cdot h}{2}$	Since they have same base & height they have same area

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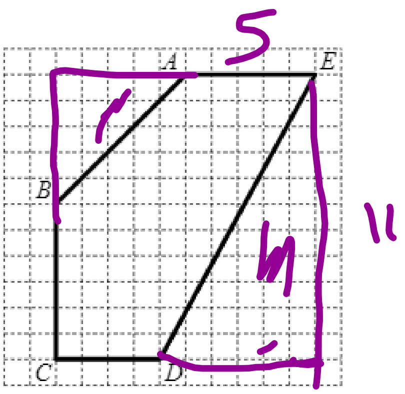
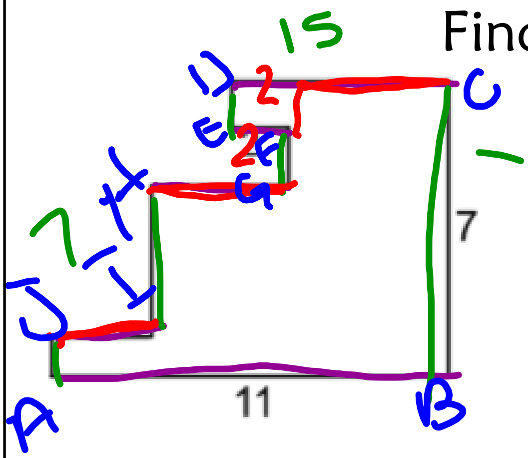


How can we find the area?

Perimeter?

Find the perimeter



$7 + 7 + 11 + 15$

40