UMTYMP Geometry Day 4

Similarity and Quiz Review

Vincent Hall Room 6

Instructor: Andrea Butler

HW Collection
Quiz info
Other concerns
Lesson on Similarity
Break
Quiz Review

Solve the proportion:

$$\frac{x}{12} = \frac{x + 8}{15}$$

Using Proportions

Similar Polygons: Two polygons with

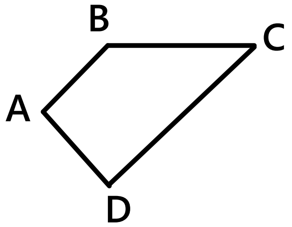
Corresponding angles that are congruent

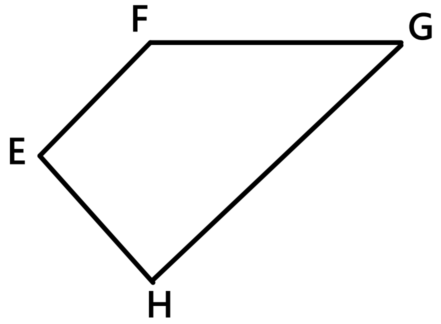
Corresponding sides that are proportional

The symbol for similarity is ~

If these two polygons are similar, we write a similarity statement:

ABCD ~ EFGH

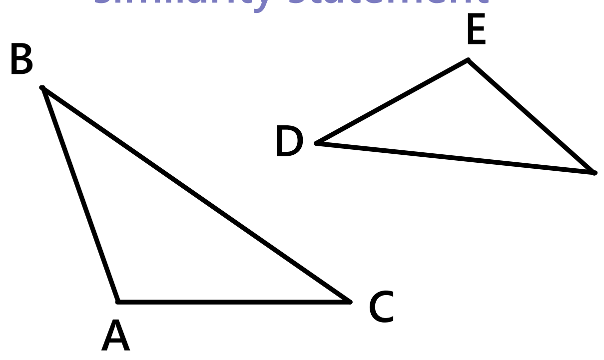




PROPORTIONALITY STATEMENTS

Given △ ABC ~△ EDF, set up a proportionality statement.

similarity statement

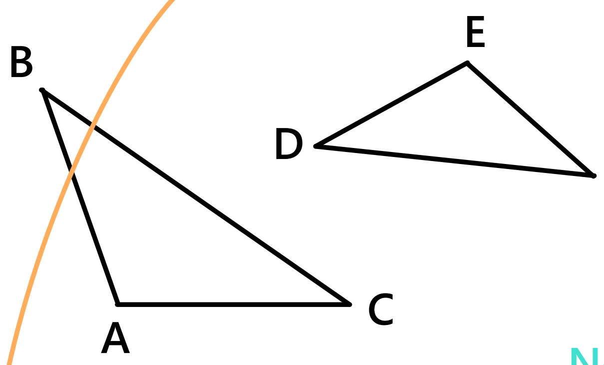


$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{CA}{FE}$$

proportionality statement

PROPORTIONALITY STATEMENTS

Given △ ABC ~△ EDF, set up a proportionality statement.



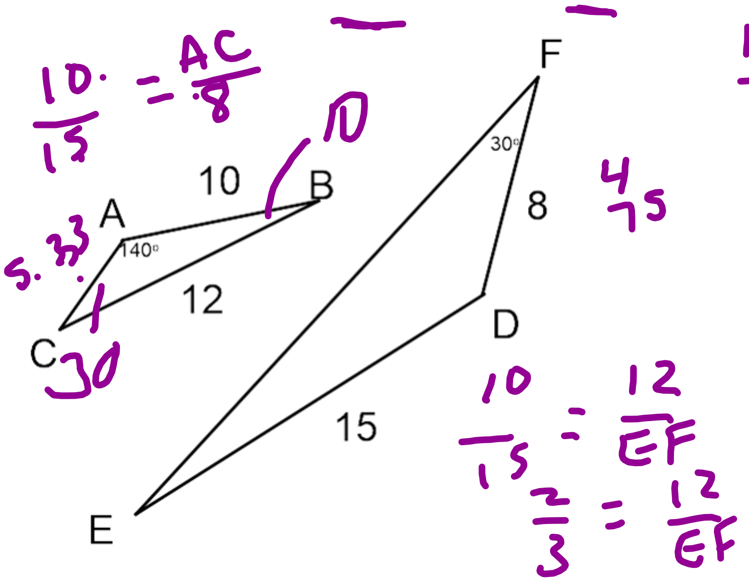
$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{CA}{FE}$$

Note: all of the segments from the same triangle go on top

You can just look at the similarity statement for the letter order!

Using Proportions

10. Given that $\triangle ABC \sim \triangle DEF$ find the following:



$m\angle C =$ 30

$m\angle D =$ 140

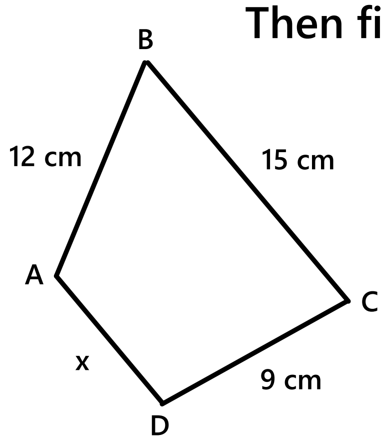
AC = 5.33

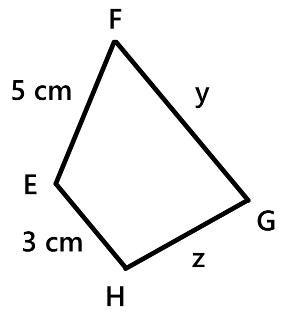
EF = 18

Using Proportions

The quadrilaterals below aresimilar. Write a similarity statement.

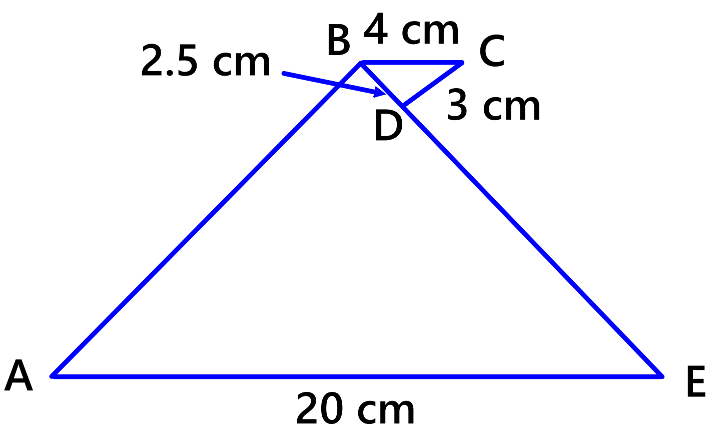
Then find the value for x, y and z.





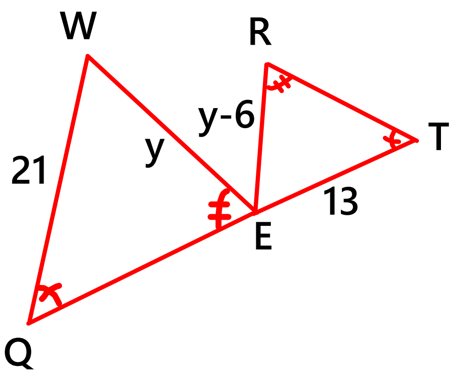
Similar Triangle Examples

Ex 1: △EBA ~ △CDB. Find the length of BE and BA.



Proportion Examples

Write a proportionality statement and solve for y.



△QEW ~ △ ______

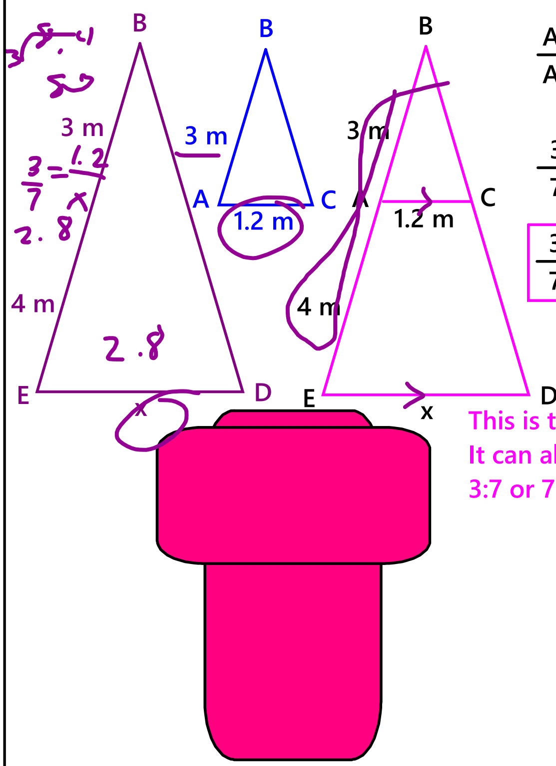
Scale Factor

The ratio of any two corresponding lengths in two similar geometric figures is called scale factor.

Different ways that scale factor can be written:
4:3, 3:4, 0.75, $1.\overline{3}$, 4/3, 3/4

Scale Factor Examples

Ex 4: Given △ABC ~ △EBD, what is the scale factor?
Find the value of x.



$$\frac{AB}{AC} = \frac{EB}{ED}$$

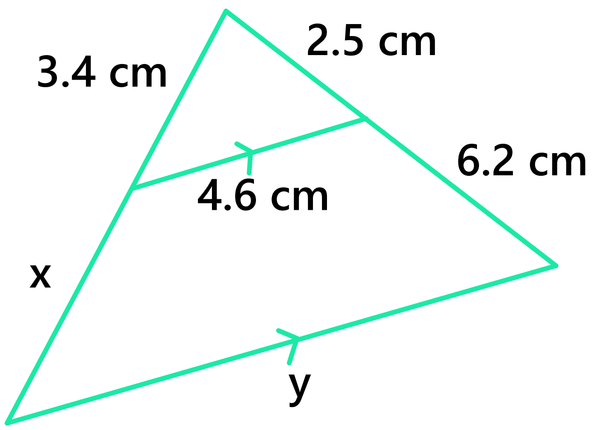
$$\frac{3}{7} = \frac{1.2}{x}$$

$$\boxed{\frac{3}{7}} = \frac{1.2}{x}$$

This is the scale factor.
It can also be written as
3:7 or 7:3 or 7/3

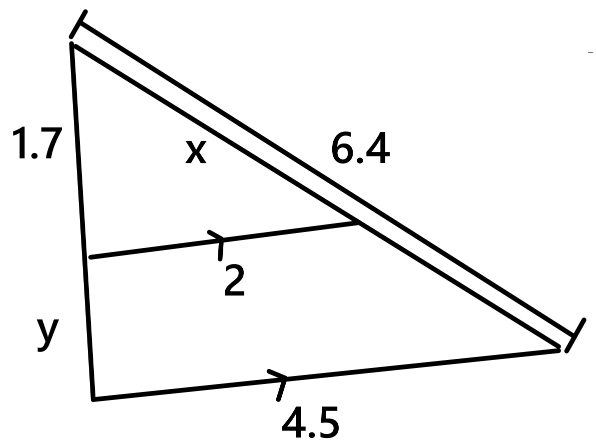
Proportion Examples

Find the variables given the two triangles are similar.



Success Criteria

Find the values of x and y.



Other Special Similarity Properties

Property 1:

If two triangles are similar, then the corresponding altitudes, corresponding medians, and corresponding angle bisectors are proportional to their corresponding sides.

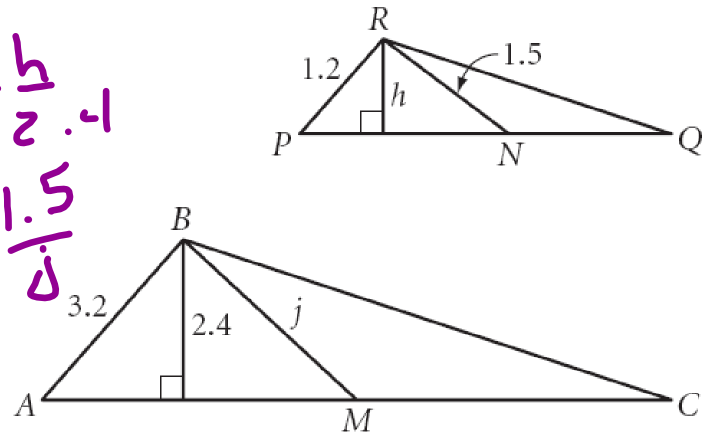
Other Special Similarity Properties

$\triangle ABC \sim \triangle PRQ$. M and N are midpoints. Find h and j.

$\frac{1.2}{3.2} = \frac{h}{2.4}$

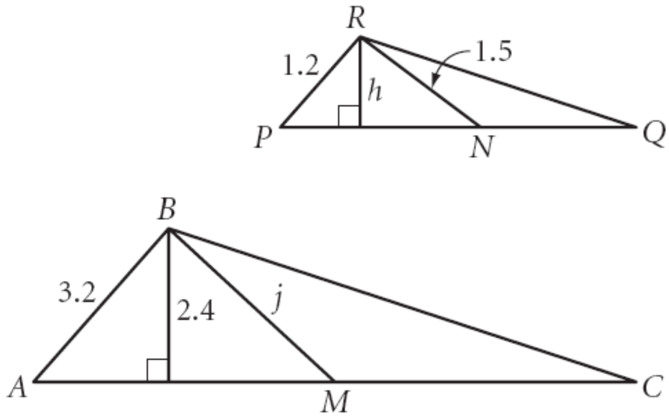
$\frac{1.2}{3.2} = \frac{1.5}{j}$

$\frac{3}{8}$



Other Special Similarity Properties

$\triangle ABC \sim \triangle PRQ$. M and N are midpoints. Find h and j.



$$\frac{3.2}{1.2} = \frac{2.4}{h}$$

$$\frac{3.2}{1.2} = \frac{j}{1.5}$$

$$j = 4$$
$$h = .9$$

Other Special Similarity Properties

Property 2:

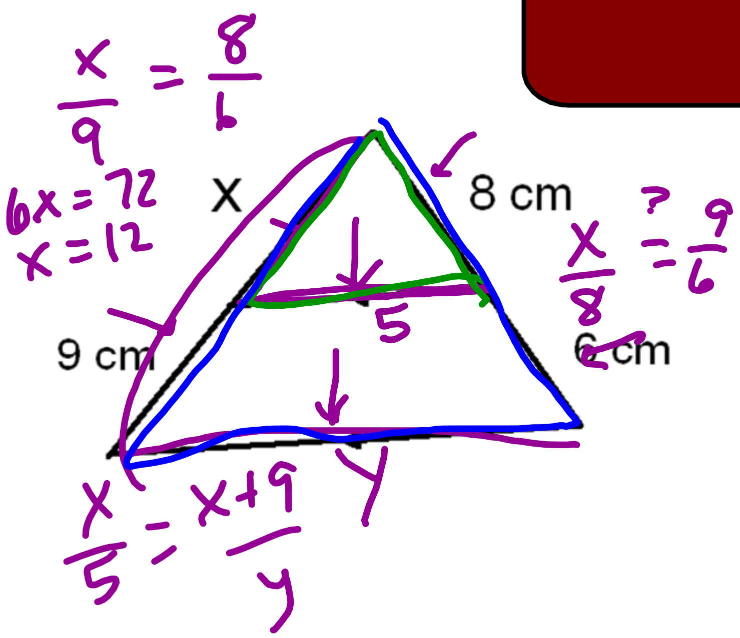
If a line parallel to one side of a triangle passes through the other two sides, then it divides them proportionally. Conversely, if a line cuts two sides of a triangle proportionally, then it is parallel to the third side.

OTHER SPECIAL SIMILARITY PROPERTIES

Find the value of x.

[redacted] or [redacted]

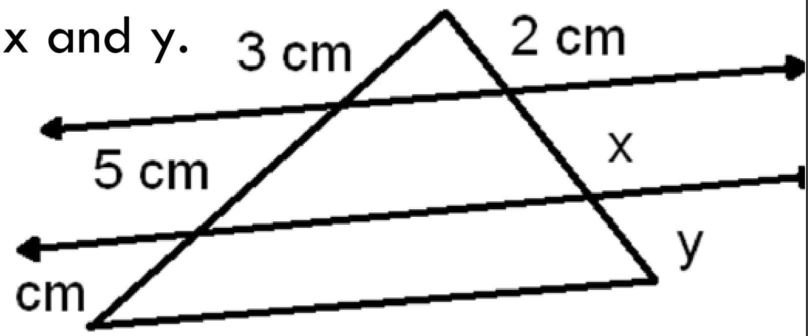
[redacted]



OTHER SPECIAL SIMILARITY PROPERTIES

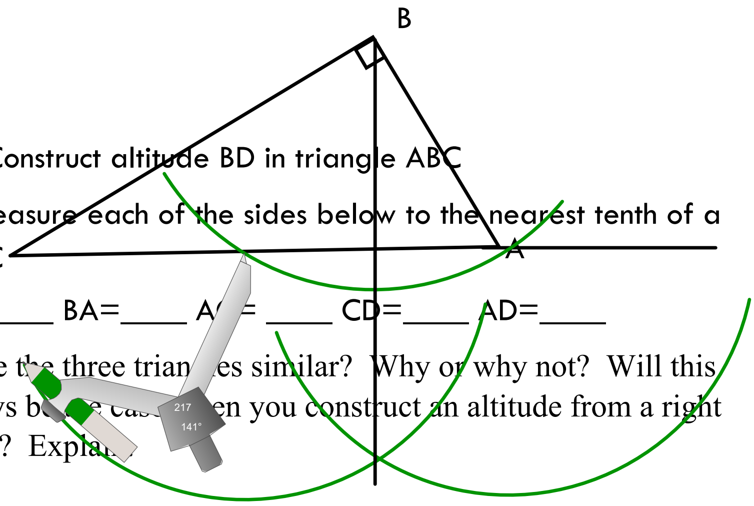
If two or more lines pass through two sides of a triangle parallel to the third side, then they divide the two sides proportionally.

Assuming the intersecting lines are parallel, find the value of x and y.



Other Special Similarity Properties

1.



1a) Construct altitude BD in triangle ABC

b) Measure each of the sides below to the nearest tenth of a cm.

CB=____ BA=____ AC= ____ CD=____ AD=____

c) Are the three triangles similar? Why or why not? Will this always be the case when you construct an altitude from a right angle? Explain.

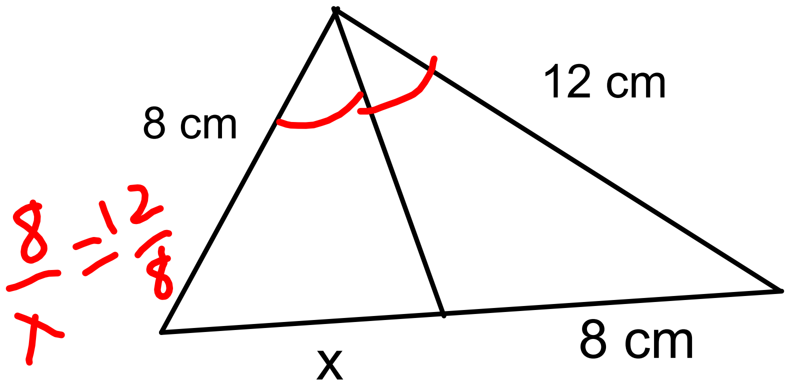
Other Special Similarity Properties

Property 3:

The angle bisector in a triangle divides the opposite side into two segments whose lengths are in the same ratio as <u>the lengths of the two sides forming the angle</u>.

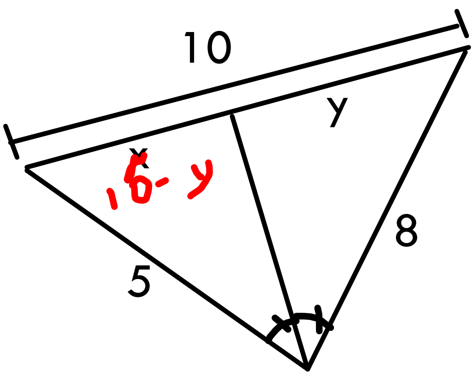
Other Special Similarity Properties

Solve for the measure of x



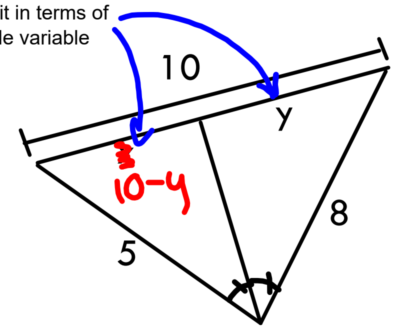
Other Special Similarity Properties

Solve for x and y



Other Special Similarity Properties

Solve for x and y



$$\frac{5}{10-y} = \frac{8}{y}$$

$$5y = 80 - 8y$$

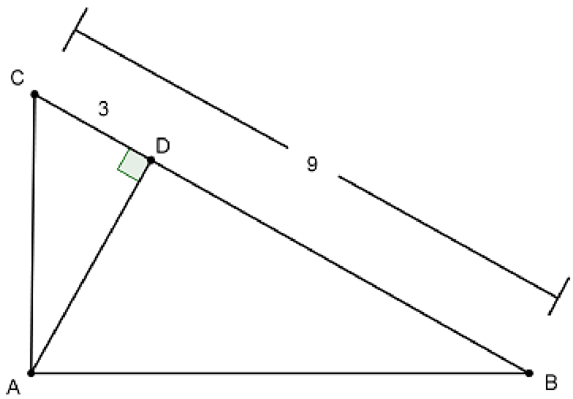
$$\frac{13y}{13} = \frac{80}{13}$$

$10 - 6.15 = 3.85 = x$

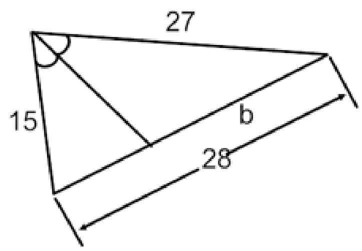
$y = 6.15$

You try!

11. $\overline{CA} \perp \overline{AB}$ CA =

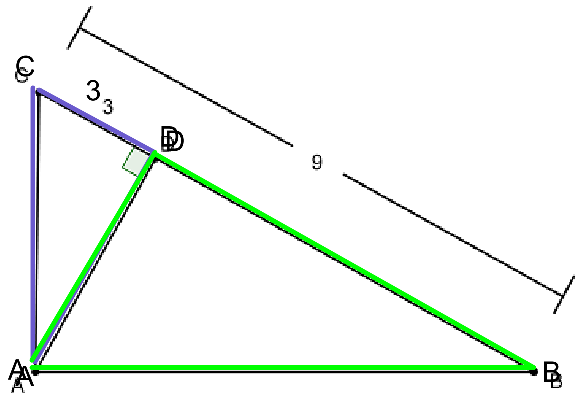


12. $b =$ _______

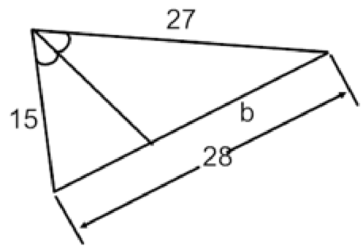


YOU TRY!

11. $\overline{CA} \perp \overline{AB}$ CA =



12. $b =$ ______



$$\frac{15}{28-b} = \frac{27}{b}$$

$$15b = 756 - 27b$$

$$42b = 756$$

$$b = 18$$

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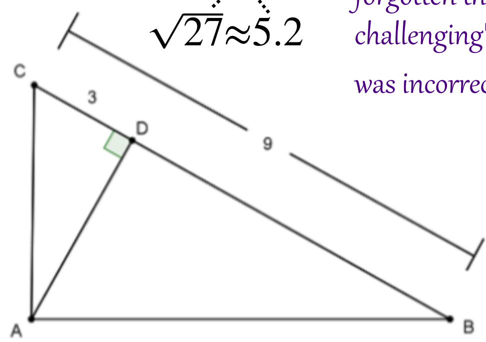
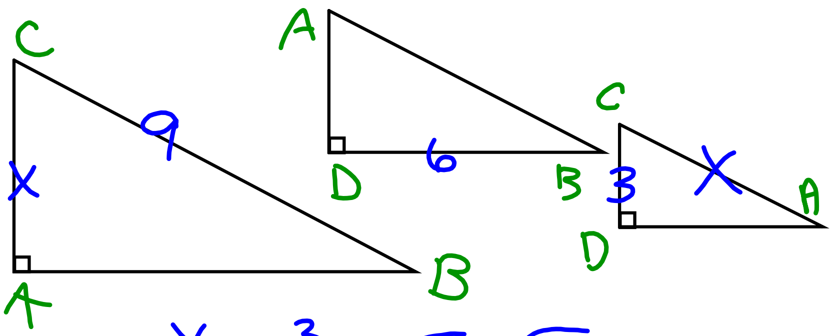
11. $\overline{CA} \perp \overline{AB}$ CA = ~~4.24~~

$\sqrt{27} \approx 5.2$

Revisiting this from yesterday...I had forgotten that this was the "very challenging" problem. (and the answer was incorrect so that didn't help :-/)





$$\frac{x}{9} = \frac{3}{x} \quad \sqrt{27} = \sqrt{x^2}$$

$$x \approx 5.$$

Similarity in the Real World

+ Finding height of an object

+ Finding distance to an object

+ Finding proportions between two things

+ Building something from a set of blueprints.

+ Drawing

+ Taking a picture

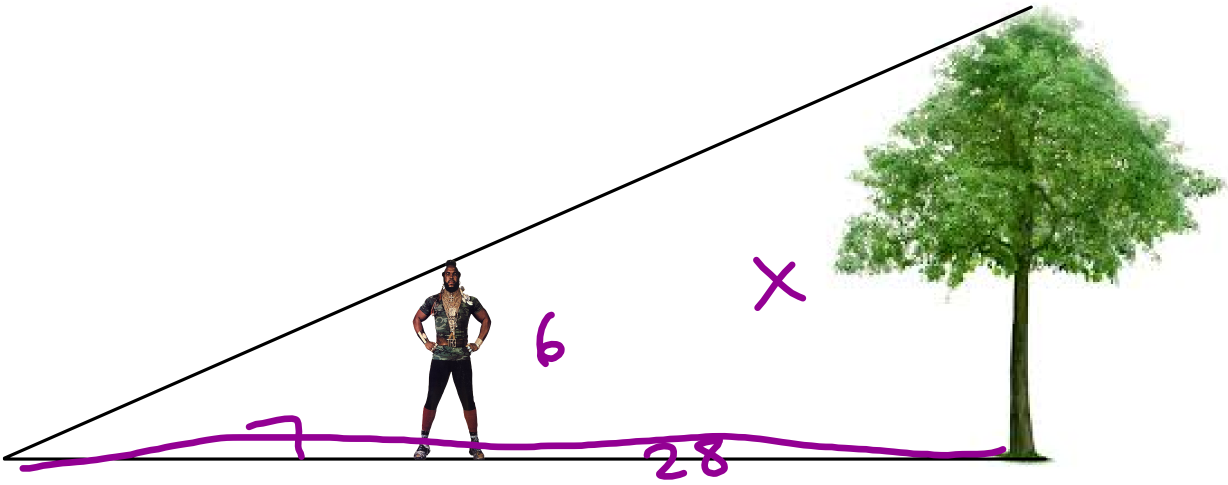
+ Any type of designer

All of these need similarity!!

Mr. Hieb is 6 ft tall casts an 84 in shadow, how tall is Mrs. Holt if at the same time her shadow is one foot shorter than his?

Similarity in the Real World

Mr. T is 6 ft tall and casts a shadow that is 7 ft long. He is standing next to a tree that is casting a 28 ft shadow. How tall is the tree?

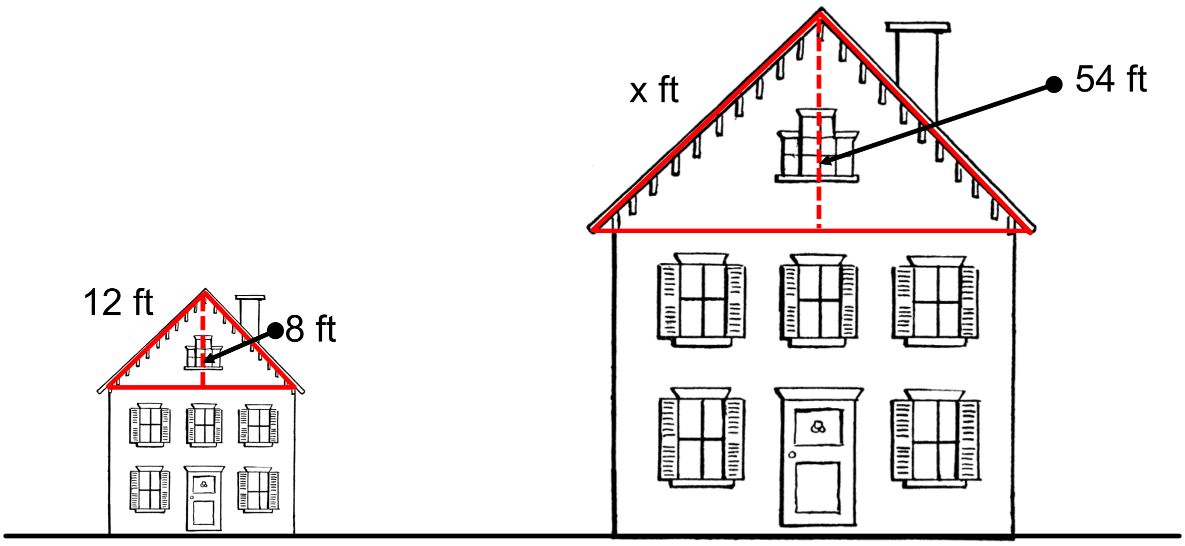


Similarity in the Real World

Kareem needed a tree in his yard cut down. The tree company asked if he knew the height of the tree. Kareem wanted to find out. He started at the base of the tree and walked along the shadow of the tree until he noticed his shadow matched up the end of the trees shadow. He knew that he was six feet tall and the distance from his feet to the end of the shadow is 8 feet. Then, he measured the shadow of the tree to be 35 ft. Use this information to find the height of the tree.

Similarity in the Real World

Ms. Dahlke and Mr. Scardigli are building houses that are proportional in size. Mr. Scardigli's house is smaller than Ms. Dahlke's. With the information given, find out how much longer Ms. Dahlke's roof is than Mr. Scardigli's roof.



Similarity in the Real World

Two ladders are leaned against a wall such that they make the same angle with the ground. The 10' ladder reaches 8' up the wall. How much further up the wall does the 18' ladder reach?

1 Make a drawing for the scenario	2 Label the important data	3 Identify what you're trying to solve for.	4 Set up your equation and solve

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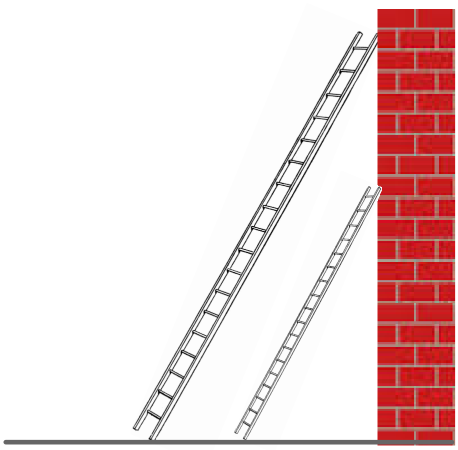

1	2	3	4
Make a drawing for the scenario	Label the important data	Identify what you're trying to solve for.	Set up your equation and solve

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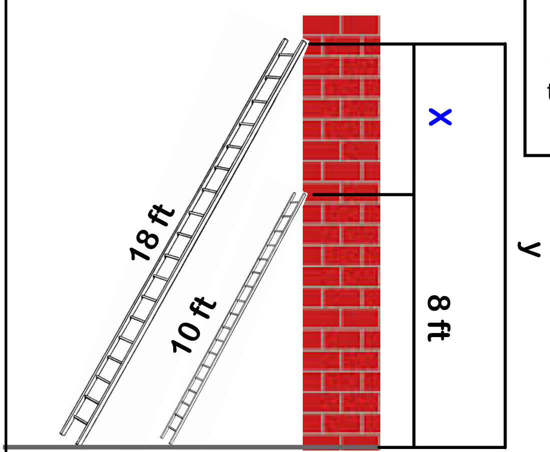

1	2	3	4
Make a drawing for the scenario	Label the important data	Identify what you're trying to solve for.	Set up your equation and solve

Similarity in the Real World

Two Triangles are similar. The sides of the first triangle are 7, 9, and 11. The smallest side of the second triangle is 21. Find the perimeter of the second triangle.

1	2	3	4
Make a drawing for the scenario	Label the important data	Identify what you're trying to solve for.	Set up your equation and solve

Similarity in the Real World

King Kong on top of the Empire State Building casts a shadow 120 meters long. The Empire State Building alone is 485 meters high and casts a shadow 97 meters long. How tall is King Kong?

1	2	3	4
Make a drawing for the scenario	Label the important data	Identify what you're trying to solve for.	Set up your equation and solve

Success Criteria Day 2

You are flying a kite and want to figure out how high it is. You tie the string to the ground and measure out 1 m and then measure that the string is 4 m high. The distance from where the string is staked to directly underneath the kite is 50 m.

Success Criteria Day 2

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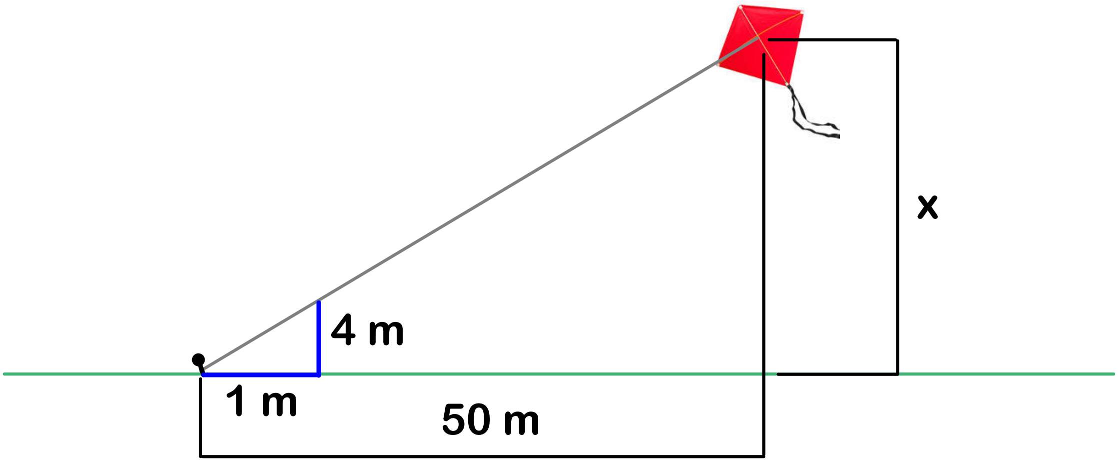


TRIANGLE CONGRUENCE

Name the conjectures that prove triangles congruent.

SSS
SAS
SAA
ASA

Name the conjectures that do not work when proving triangles congruent.

AAA SSA

CONGRUENCE VS. SIMILARITY

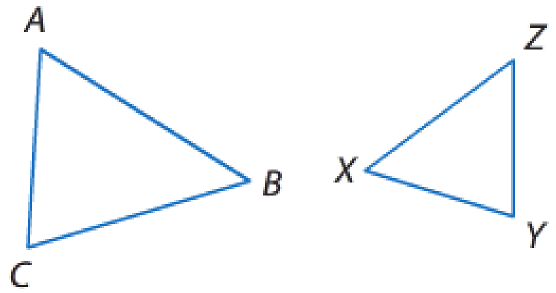
What are the difference between congruent triangles and similar triangles?

Congruent	Similar
All ≅ sides All ≅ angles	All ≅ angles proportional sides

Triangle Similarity

In order to prove two figures are similar, you must show:

- corresponding sides are proportional
- corresponding angles are congruent



$\angle A \cong \angle Z$, $\angle B \cong \angle Y$, $\angle C \cong \angle X$;

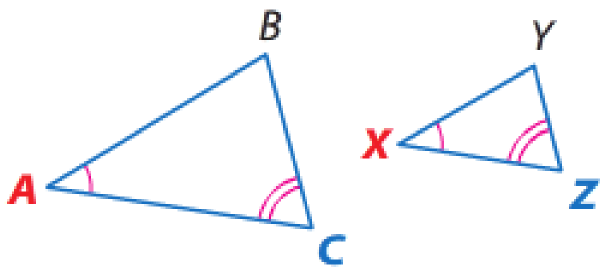
$$\frac{AC}{ZX} = \frac{BC}{YX} = \frac{AB}{ZY}$$

$\triangle ABC \sim \triangle ZYX$

But today we're going to learn a short cut for triangles
(just like we did with triangle congruency)

AA Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.



If $\angle A \cong$ ___ and $\angle C \cong$ ___

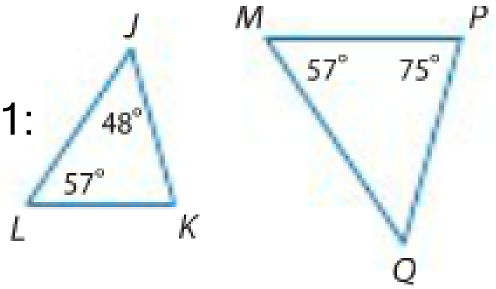
then $\triangle ABC \sim \triangle$ ____

This is what's called a "similarity statement"

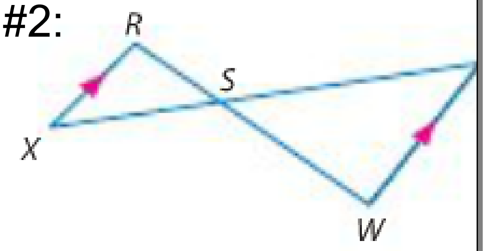
Examples

Are these triangle similar? If so, write a similarity statement and explain your reasoning.

Ex #1:

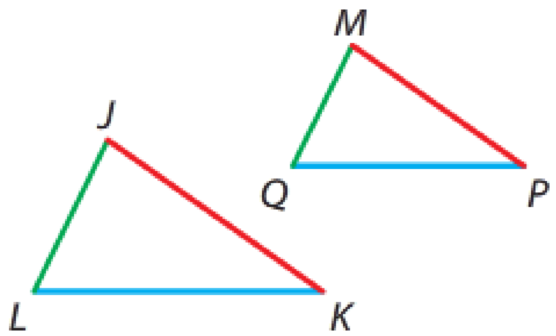


Ex #2:



SSS Similarity Theorem

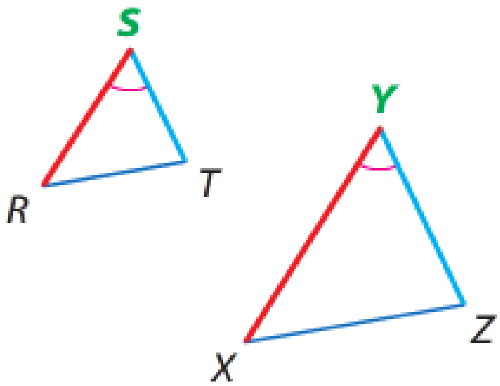
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.



If $\frac{\_\_}{MP} = \frac{KL}{\_\_} = \frac{\_\_}{QM}$, then $\triangle JKL \sim \triangle MPQ$.

SAS Similarity Theorem

If the lengths of two sides of one triangle are proportional to the lengths of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.



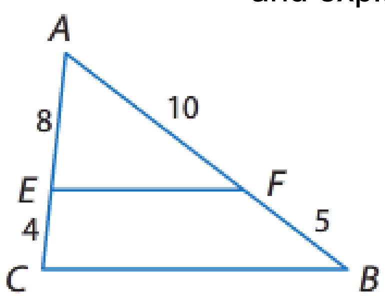
If $\frac{RS}{\quad} = \frac{ST}{\quad}$ and $\angle S \cong$ ____ then

$\triangle RST \sim \triangle XYZ.$

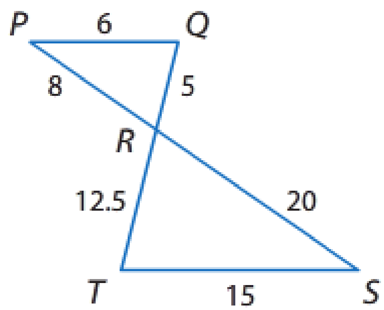
A couple more examples

Are these triangles similar? If so, write a similarity statements and explain your reasoning.

Ex #1:



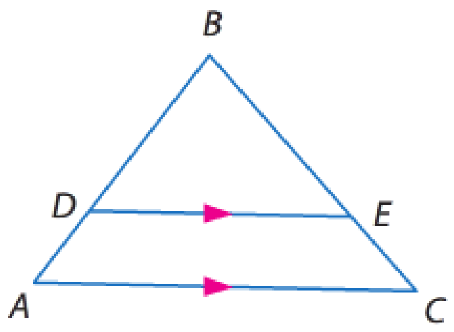
Ex #2:



PROOFS

Given: $\overline{DE}$ is parallel to $\overline{AC}$

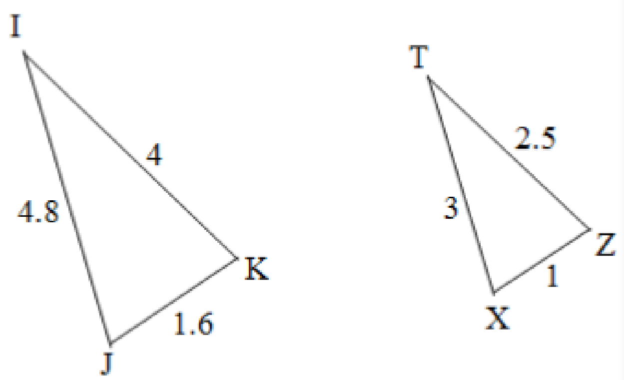
Prove: $\triangle ABC \sim \triangle DBE$



Statements	Reasons

Given: the segement lengths shown

Prove: $\triangle IJK \sim \triangle$___



Statements	Reasons

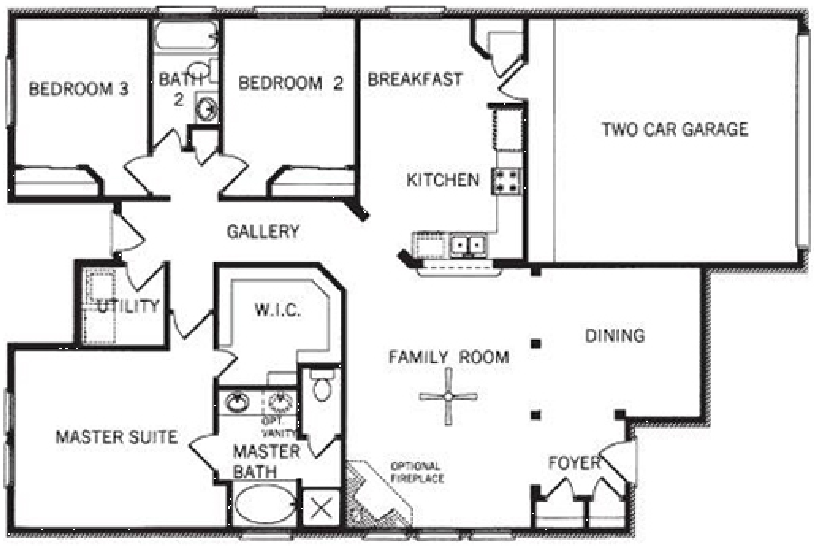
8. On the floor plan below, 2 cm represents 10 feet. *(www.brightonhomes.com)*

(a) What are the actual dimensions of the master suite not including the walk in closet (WIC) and master bath?

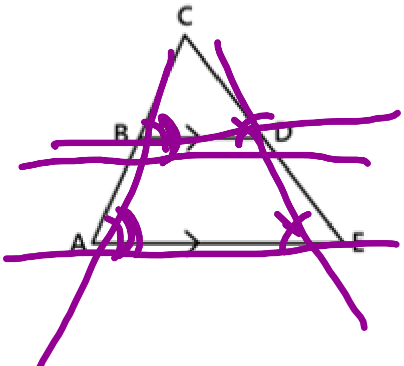
(b) What are the actual dimensions of the combined kitchen and breakfast area?

(c) What are the dimensions of the dining area?

(d) How many times larger is the actual house compared to this floor plan?

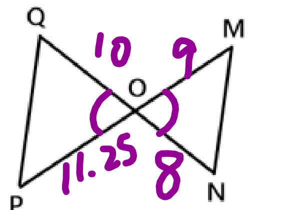


11. Complete the proof.
Given: $\overline{AE} \parallel \overline{BD}$
Prove: $\Delta ACE \sim \Delta BCD$



Statements	Reasons
1. $\overline{AE} \parallel \overline{BD}$	1. given
2. ∠CBD ≅ ∠CAE	2. Corresponding angles
3. ∠CDB ≅ ∠CEA	3. corresponding angles
4. △ACE ~ △BCD	4. AA similarity

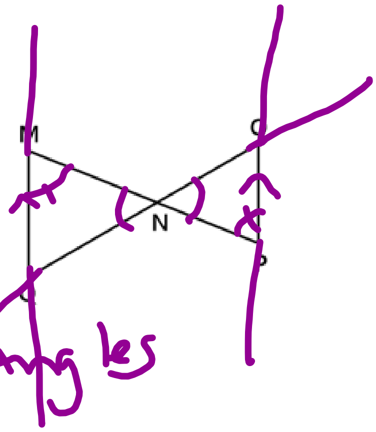
12. Write a proof for the following
Given: NO = 8 cm, OQ = 10 cm
MO = 9 cm, OP = 11.25 cm
Prove: $\Delta MON \sim \Delta POQ$



Statements	Reasons
1. N	1. given
2. ∠QOP ≅ ∠NOM	2. vertical angles
3. $\frac{MO}{OP} = \frac{NO}{OQ}$	3. sides proportional
4. △MON ~ △POQ	4. SAS Similarity

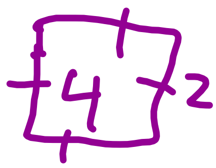
$\frac{9}{11.25} \stackrel{?}{=} \frac{8}{10}$

13. Given: $\overline{MQ} \parallel \overline{OP}$ NO
Prove: $\Delta MNQ \sim \Delta PON$

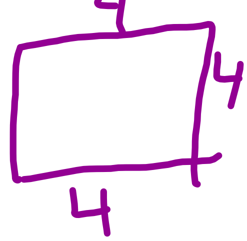


Statements	Reasons
1. $\overline{MQ} \parallel \overline{OP}$	1. given
2. ∠MNQ ≅ ∠PNO	2. Vert. Angles
3. ∠QMN ≅ ∠OPN	3. AIA
4. ΔMNQ ~ ΔPON	4. AA Sim

Ratios of areas!

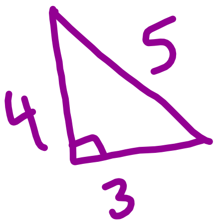


Sides $\frac{2}{4}$ Areas $\frac{4}{16}$

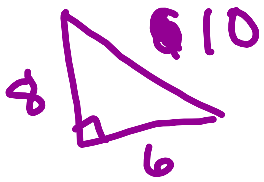


$\frac{1}{2}$ $\frac{1}{4}$

Sides $\frac{m}{n}$ Areas $\frac{m^2}{n^2}$ ★



$\frac{1}{2} \cdot 3 \cdot 4 = 6$



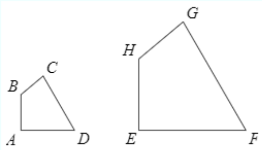
$\frac{1}{2} \cdot 8 \cdot 6 = 24$

k^2

Recap from your book:

Definition: Two figures are **similar** if one is simply a blown-up, and possibly rotated and/or flipped, version of the other.

Important: Corresponding angles in similar figures are equal, and the ratio of the lengths of corresponding sides of similar triangles is always the same.



In similar quadrilaterals $ABCD$ and $EHGF$, we have $\angle A = \angle E$, $\angle B = \angle H$, $\angle C = \angle G$, and $\angle D = \angle F$. We also have

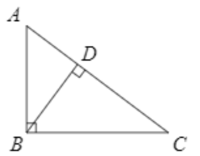
$$\frac{AB}{EH} = \frac{BC}{HG} = \frac{CD}{GF} = \frac{DA}{FE} = \frac{AC}{EG} = \frac{BD}{HF}.$$

We denote these figures as similar by writing $ABCD \sim EHGF$.

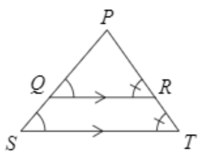
There are three main ways to show that two triangles are similar:

- **AA Similarity**. If two angles of one triangle equal two angles of another, then the triangles are similar. This is by far the most commonly used method to prove two triangles are similar. (Section 5.2)
- **SAS Similarity**. If two sides in one triangle are in the same ratio as two sides in another triangle, and the angles between the sides in each triangle equal each other, then the triangles are similar. (Section 5.3)
- **SSS Similarity**. If each side of one triangle is the same constant multiple of the corresponding side of another triangle, then the triangles are similar. (Section 5.4)

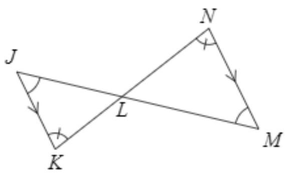
Parallel lines and perpendicular lines are clues to look for similar triangles. Three very common set-ups that contain similar triangles are shown below.



$\triangle ABC \sim \triangle BDC \sim \triangle ADB$



$\triangle PQR \sim \triangle PST$

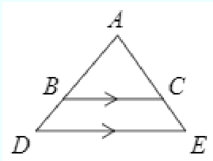


$\triangle JLK \sim \triangle MLN$

Important:

If $\overline{BC} \parallel \overline{DE}$ and $\overleftrightarrow{BD}$ and $\overleftrightarrow{CE}$ meet at A as shown, then

$$\frac{AB}{BD} = \frac{AC}{CE}.$$



Important:

If two triangles are similar such that the sides of the larger triangle are k times the sides of the smaller, then the area of the larger triangle is k^2 times that of the smaller.

This relationship holds for any pair of similar figures, not just for triangles.

Problem Solving Strategies

Concepts:

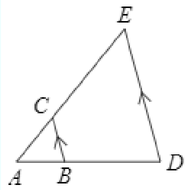
- When you're stuck on a problem, ask yourself, 'What piece of information have I not used?'
- In many problems, there's more than meets the eye. Extending segments that seem to end abruptly (particularly in the middle of a triangle) can often yield quick solutions.
- When stuck on a problem, try solving an easier related problem. For constructions, useful easier related problems often involve relaxing one of the constraints of the problem.
- Consider using similar triangles in problems involving ratios of segment lengths.

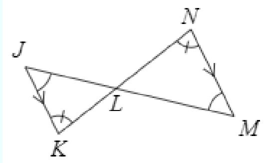
Things To Watch Out For!

WARNING!!

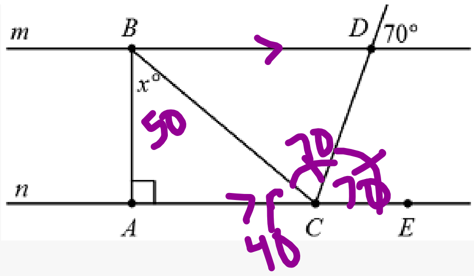
⊘

Below are shown two common situations that lead to mistakes. The diagram on the left may lead you to write '$\triangle ABC \sim \triangle ADE$, so $AB/BD = BC/DE$.' The one on the right might lead to '$\triangle JKL \sim \triangle NLM$, so $JL/NL = KL/ML$.' Both of these are **incorrect**! Make sure you see why!



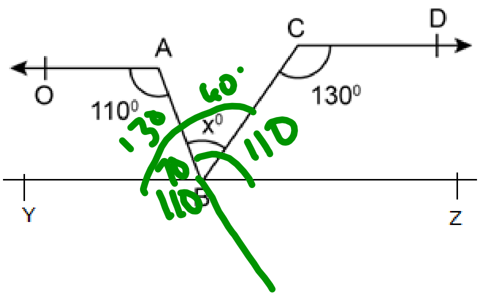


Constructions!!!



In the figure above, $m \parallel n$ and $\overline{CD}$ bisects $\angle BCE$. Which of the following is the value of x?

| In the figure, AO and CD are parallel to YZ. Find the value of x.



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Recap from our course so far:

Points, Lines, and Planes

Angles

Congruent Triangles

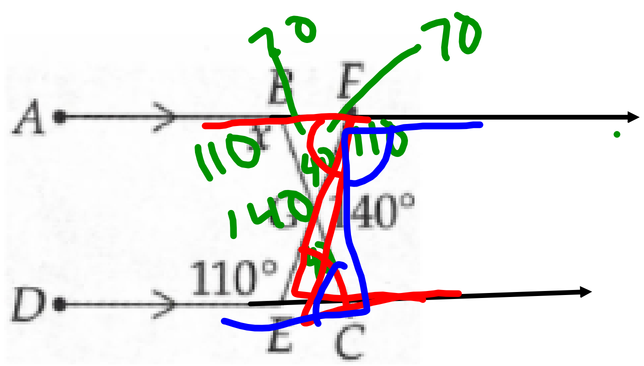
Perimeter and Area

GR GR

proofs SSS, SAS, ASA, SAA

CPCTC - only after △s ≅

Solve for 'x'.

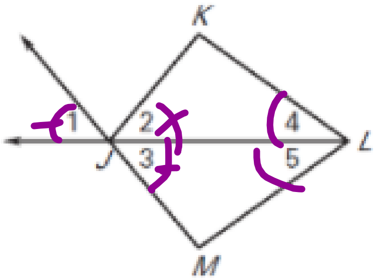


$\angle A = x - 2y$
$\angle B = 3x + 5y$
$\angle C = 5x - 3y$

Find ‘x’. What are the possible positive values of ‘y’ if one of the angles is 10°?

GIVEN: $\angle 2 \cong \angle 1$, $\angle 4 \cong \angle 5$

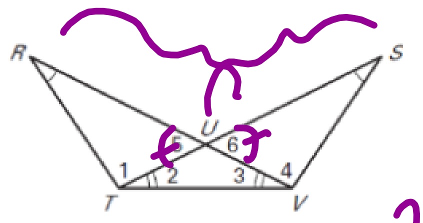
PROVE: $\overline{KL} \cong \overline{ML}$



Reason	Stmt
1. $\angle 2 \cong \angle 1$, $\angle 4 \cong \angle 5$	1. given
2. $\angle 1 \cong \angle 3$	2. Vertical angles
3. $\overline{LJ} \cong \overline{LJ}$	3. Reflexive
4. $\triangle JKL \cong \triangle JML$	4. ASA
5. $KL \cong ML$	5. CPCTC

GIVEN: $\angle R \cong \angle S$, $\angle 2 \cong \angle 3$

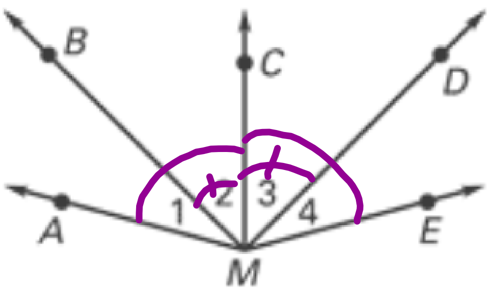
PROVE: $\overline{RU} \cong \overline{SU}$



S. | R.

1. ∠R ≅ ∠S, ∠2 ≅ ∠3 | 1. given
2. ∠5 ≅ ∠6 | 2. vert.
3. ∠1 ≅ ∠4 | 3. ∠R + ∠5 ≅ ∠S + ∠6 so 180 = ∠R + ∠5 ≅ ∠S + ∠6
4. TV ≅ TV | 4. Ref

Prove using a 2-column proof that if *CM* bisects angles *BMD* and *AME*, then angles *1* and *4* are congruent.

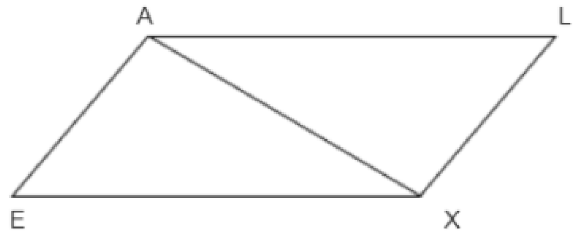


R | S

1. CM bisects BMD and AME | 1. given
2. ∠1 + ∠2 = ∠3 + ∠4, ∠2 ≅ ∠3 | 2. defn bisect
3. ∠1 + ∠2 = ∠3 + ∠4, ∠1 + ∠2 = ∠2 + ∠4 | 3. substitut
4. ∠1 ≅ ∠4 | 4. subtraction

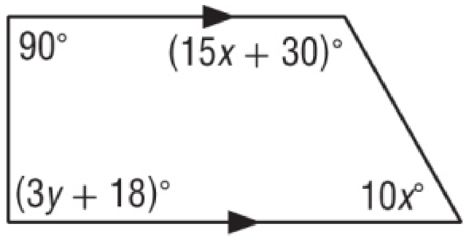
Given: *EAX* and *LXA* are right angles
EA = *LX*

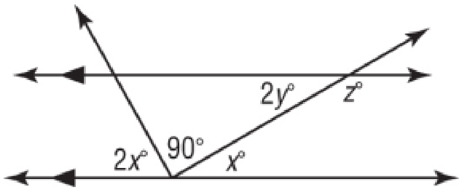
Prove: *AL* // *EX* (prove they are parallel)

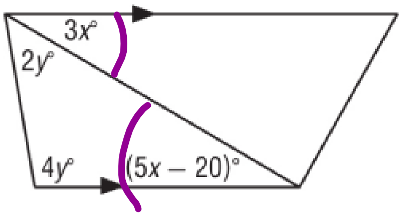


The area of a triangle is 27ft^2. If the height is 3 times the length of its base, find the height and base of the triangle.

Find the area of an equilateral triangle with a perimeter of 24in. ALSO What is the simplified area formula for any equilateral triangle?



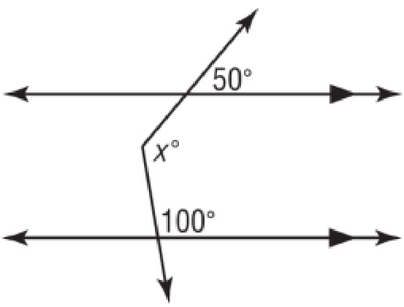


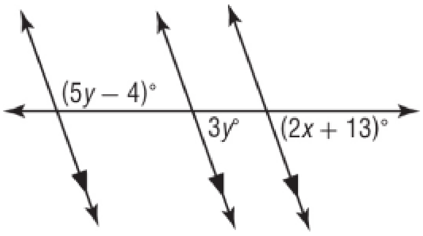


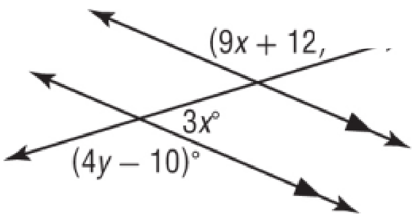
$3x = 5x - 20$

$-2x = -20$

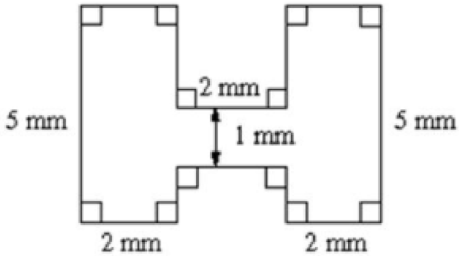
$x = 10$



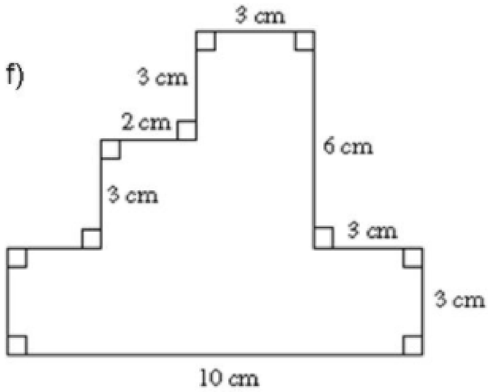




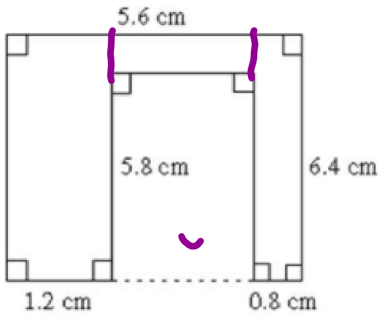
e)


5 mm
2 mm
1 mm
5 mm
2 mm
2 mm

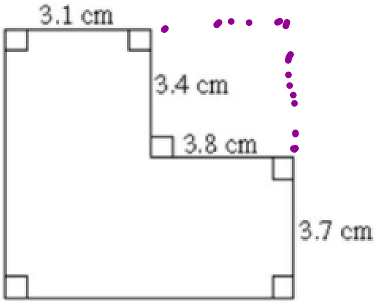
f)


3 cm
3 cm
2 cm
6 cm
3 cm
3 cm
3 cm
10 cm

g)


5.6 cm
5.8 cm
6.4 cm
1.2 cm
0.8 cm

h)


3.1 cm
3.4 cm
3.8 cm
3.7 cm